The design and welfare implications of mandatory pension plans

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Abstract: In a rich, calibrated life-cycle model, we show that well-designed mandatory pension plans significantly improve the welfare of individuals procrastinating on savings or not investing in stocks, and even improve rational individuals' welfare through a return tax advantage and fair annuitization. For a group of heterogeneous savers, in terms of preferences and sophistication, the best plan has contributions of 9% of income from age 30, a glidepath investment strategy, payouts following a variable lifelong annuity, and options to choose a different investment strategy and to de-select the annuitization feature. This plan generates an average welfare gain of $233,000 per individual.

Keywords: Retirement saving, life cycle, consumption, investment, annuitization, welfare, procrastination, non-participation.

JEL subject codes: D91, G11, D14, E21, J32.

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1 Introduction

Large population groups build up insufficient savings for retirement. Close to half of adult Americans do not have access to a workplace retirement saving plan, and more than half worry that they will not have enough money for retirement.[1] In a large sample of U.S. workers enrolled in a defined contribution pension plan, Gomes, Hoyem, Hu, and Ravina (2020) estimate that 75% of individuals save too little for retirement. Apparently, a large share of households procrastinate on retirement saving and thus end up with a highly unbalanced life-cycle consumption profile. Retirement saving is becoming even more challenging due to the low current interest rates and expected returns, the predicted rise in longevity, the possible cuts in Social Security payouts, the soaring health care costs, and the increase in self-employed and gig workers.

Evidently, mandatory or auto-enrollment pension plans can reduce the under-saving problem and thus also reduce the demand for tax-financed benefits to retirees (e.g. Statman, 2013). Mandatory defined contribution saving plans already exist for the majority of workers in many countries including the U.K. and the three countries with the World's best pension systems, namely the Netherlands, Denmark, and Australia (according to the 2019 Mercer Global Pension Index). In the U.S., several states are currently introducing or expanding such programs, while some economists and politicians push for a universal mandatory retirement saving program, e.g., the plan promoted by Ghilarducci and James (2018). Opponents point to a general aversion against government intervention in personal decisions and, more specifically, that mandatory programs will harm rational individuals capable of accumulating sufficient savings on their own.[2]

Based on a rich life-cycle model calibrated to U.S. data, we show that a well-designed mandatory pension plan greatly improves the welfare of "irrational" individuals by bringing their level and life-cycle profile of consumption closer to the rational ideal. The welfare gains are not only due to a more balanced life-cycle consumption-saving profile. As documented in an extensive empirical literature,

[1] See 2018 Bureau of Labor Statistics at https://www.bls.gov/ncs/ebs/benefits/2018/benefits_retirement.htm and a 2019 Gallup survey at https://news.gallup.com/poll/249164/americans-feel-generally-positive-own-finances.aspx.

[2] Small businesses oppose plans requiring them to handle their employees' saving plans, but several universal plan proposals suggest detaching retirement savings from employers as is done in other countries.

many households invest very little, if anything, of their wealth in stocks, in contrast to the prescriptions of theoretical models—the so-called stock market participation puzzle. If the mandatory plan features a default glide-path investment strategy with a relatively large fraction of savings invested in stocks, such households can expect to build larger savings and enjoy more consumption. The annuitization puzzle is also well documented: the vast majority of households do not annuitize savings entering retirement and thus have either to stretch their savings to cover consumption over the maximum life time or to risk outliving their wealth. An automatic annuitization feature can easily be build into a broad or universal pension plan. Hence, a well-designed mandatory pension plan can simultaneously mitigate the under-saving problem, the stock market participation puzzle, and the annuitization puzzle.

Consider, for example, a 25-year old individual with initial wealth and life-cycle income profile similar to the median U.S. worker, a relative risk aversion (RRA) of 4, and an elasticity of intertemporal substitution (EIS) of 1/4. How is the welfare of the individual affected by introducing a mandatory pension plan? Suppose the plan involves (i) contributions equal to 9% of income from age 30 to retirement at age 67, (ii) a glidepath investment policy with the stock weight starting at 100%, then sloping down to 40% during the last 15 years before retirement, and staying at 40% through retirement, (iii) tax-exempt returns on pension savings, and (iv) automatic annuitization. Compared to the case with only private savings, we find that this plan leads to a welfare gain of 11.5% for an individual not privately investing in stocks, but otherwise making rational decisions. In present value terms, the gain corresponds to around $170,000. Suppose instead that the individual can invest privately in stocks, but procrastinates on savings to an extent that she expects to build up only around 1/3 of the retirement savings that a fully rational individual does (about $185,000 instead of $522,000). The mandatory plan improves the welfare of the procrastinating individual by 44.0% or roughly $648,000 in present value terms.

The welfare of rational individuals deteriorate, other things equal, when imposing constraints in the form of mandatory savings invested according to a prespecified asset allocation strategy. However, for plans with moderate contribution rates, the welfare reduction is often small as the rational individual can adjust private saving and investment decisions accordingly. A mandatory pension plan can positively affect the welfare of rational individuals through two channels: a lower taxation of returns on pension investments than private investments (as seen in practice) and an access to annuitization at a better price than what is available in the current annuity market.[3] We show that some simple tax-exempt, auto-annuitization plans improve the welfare of a range of rational individuals. For example, the specific plan described above improves the welfare by 4.5% for a rational individual still assuming RRA=4 and EIS=1/4. Most of this gain is due to the automatic and fairly priced annuitization.

A broad mandatory plan covers a large number of heterogeneous individuals. We consider 27 types of individuals generated by combining three values of RRA (2, 4, and 6), three values of EIS (1/6, 1/4, and 1/2, thus including different timing preferences), and three levels of sophistication (rational, procrastinator, and stock avoider). We look for mandatory plan designs leading to the largest average welfare gain across the 27 types, conditional on positive gains for all 27. The main challenge is to please rational individuals having both a low RRA and a low EIS as they dislike plans with substantial mandatory savings. Hence, we consider sweetening the plan by embedding two options that are particularly valuable for such individuals. One option is to replace the default investment policy by a more aggressive one. The other is to opt out of the annuitization feature, which only individuals with low RRA and EIS will choose to do. Without such options, the contribution rate would have to be lowered to ensure support from the rational participants with low RRA and EIS but that would reduce the welfare gains for the irrational participants. According to our criterion and quantitative analysis, the plan outlined above is optimal, i.e. 9% contributions from age 30, a glidepath investment policy, lifelong annuity-style payouts, with the two options added. This plan brings welfare gains to all 27 participant types with the average gain being 15.8% or about $233,000 in present value terms.

Our main analysis assumes Social Security retirement benefits at the current level. However, according to the April 2020 report of the Social Security and Medicare Boards of Trustees, the system will experience net cash outflows from 2021, and by 2035 asset reserves are depleted and benefits have to be cut by 24% to make the system sustainable.[4] Alternatively, the retirement age or the pay-roll tax

[3] On the investment side, positive welfare effects can also come from the pension fund generating a better pre-tax risk-return tradeoff by having access to better diversification, additional asset classes (commercial real estate, infrastructure, foreign investments), and maybe superior asset selection skills.

[4] Source: https://www.ssa.gov/OACT/TR/2020/index.html, accessed on August 7, 2020.

rate financing the system would have to increase substantially, or the government would have to subsidize the system directly. The economic slowdown following the 2020 pandemic and the projected increased budget deficit and debt over the coming decades put further stress on the Social Security system.[5] With a 24% cut in Social Security, we find that contributions from age 30 are still optimal, but the contribution rate should be raised from 9% to 10%. The welfare gain induced by the mandatory pension scheme increases for all types of participants considered, with the average gain going from 15.8% to 17.8%. As expected, a well-designed mandatory pension scheme is even more appreciated if Social Security benefits are reduced.

Our quantitative analysis builds on a rich life-cycle model of individual consumption and saving decisions with uncertain labor income, mortality risk, Social Security benefits, possible out-or-pocket medical costs, and both a riskfree and a risky asset. We derive the optimal decisions and lifetime utility of a given individual both in the case without and the case with a specific mandatory saving plan, from which we quantify the plan's impact on the individual's welfare. We can then search for an optimal plan design in terms of contributions before retirement, investments of retirement savings, and payout in retirement—taking into account that the individuals optimally manage private savings and investments, possibly with restrictions reflecting a lack of financial rationality or sophistication.

We are not aware of other studies performing a similar welfare and design analysis of mandatory saving plans. The most closely related paper is by Dahlquist, Setty, and Vestman (2018). Motivated by the Swedish pension system, they determine the optimal default stock-bond asset allocation strategy (among four) of a pension plan in a setting where the investor can—at a certain cost and only at age 25—actively switch to any asset allocation strategy. The investor's private participation in the stock market is also an active decision with an associated entry cost. The authors allow for heterogeneous costs across investors to match the observed active decisions of Swedish investors. Throughout, the authors assume a given contribution rate and do not consider the optimal level of the rate (or from which age it should apply). They ignore taxes, although returns in many countries are differentially taxed depending on returns being made inside or outside retirement saving plans.

[5] Source: http://www.crfb.org/papers/updated-budget-projections-show-fiscal-toll-covid-19-pandemic, accessed on August 7, 2020.

In contrast, we incorporate taxes and quantify the welfare implications of mandatory schemes, and we search for both the optimal contribution policy, investment policy, and payout policy of the pension fund.

Our paper builds upon well-known life-cycle models (e.g. Viceira, 2001; Cocco, Gomes, and Maenhout, 2005; Gomes and Michaelides, 2005) that feature consumption and asset allocation decisions under mortality risk and unspanned stochastic labor income. These models conclude that most individuals should invest all their savings in stocks early in life and then gradually replace stocks by bonds.[6] Our model adds a mandatory illiquid pension scheme, which complicates the solution considerably as the optimal private decisions depend on the accumulated pension savings in addition to the level of labor income and private savings. We consider Epstein-Zin preferences separating attitudes towards risk and intertemporal variations. For a given design of the mandatory pension scheme, we solve this rich dynamic optimization problem for the optimal private consumption-investment decisions and derive the corresponding lifetime utility. We do this both for the fully rational, savvy investor and irrational individuals procrastinating on saving or having limited investment skills. Of course, solving the individual's optimization problem is already important for determining the extent to which an individual with a certain mandatory pension plan should build up additional liquid savings and how they should be invested. Then we search for fund designs leading to a higher lifetime utility than in the absence of a mandatory plan, and we also identify the optimal pension fund design for a given set of individual characteristics and skills.

Only few existing papers in the life-cycle literature explicitly model an illiquid pension account. Campbell, Cocco, Gomes, and Maenhout (2001) assume a predetermined, constant contribution rate and fund asset allocation, and derive the individual's optimal consumption and private investments over the life cycle. They compare welfare and optimal individual decisions for two fund allocation strategies, namely (i) 100% in the riskfree asset vs. (ii) 50% in stocks, 50% in the riskfree asset.

[6]The canonical life-cycle model has been extended to labor supply flexibility (Bodie, Merton, and Samuelson, 1992), housing (Cocco, 2005; Fischer and Stamos, 2013; Corradin, Fillat, and Vergara-Alert, 2014), time-varying investment opportunities (Koijen, Nijman, and Werker, 2010; Munk and Sørensen, 2010; Lynch and Tan, 2011), unemployment risk (Bremus and Kuzin, 2014; Branger, Larsen, and Munk, 2019), income-stock market co-integration (Benzoni, Collin-Dufresne, and Goldstein, 2007), investments in annuity products (Horneff, Maurer, and Rogalla, 2010a; Koijen, Nijman, and Werker, 2011), habit formation in preferences (Gomes and Michaelides, 2003; Polkovnichenko, 2007; Kraft, Munk, and Wagner, 2018), and stock market entry/participation costs (Fagereng, Gottlieb, and Guiso, 2017).

We take the analysis a large step further by deriving the optimal combination of contribution rate and fund allocation strategy, and we consider how the pension savings are optimally paid out in retirement. As we do, Blake, Wright, and Zhang (2014) investigate the optimal contribution rate and stock-bond allocation of the pension plan. However, they ignore the possibility of free savings outside the plan, which fixes consumption at a fraction of current income and thus prevents consumption smoothing. Furthermore, they disregard bequests and taxes. Feldstein (1985), İmrohoroğlu, İmrohoroğlu, and Joines (2003), and Cremer, De Donder, Maldonado, and Pestieau (2008) determine the optimal level of Social Security benefits or public pensions in macro-style equilibrium models.[7]

The behavioral household finance literature has documented that the financial decisions of many individuals deviate systematically from what standard theoretical models prescribe, cf. the surveys by Campbell (2006; 2016), Guiso and Sodini (2013), and Beshears, Choi, Laibson, and Madrian (2018). Some general findings are that many households invest a too small fraction of wealth (if any) in risky assets, are under-diversified, and obtain poor investment returns (Barber and Odean, 2000; Calvet, Campbell, and Sodini, 2007; Campbell, Ramadorai, and Ranish, 2019).[8] Many individuals seem to procrastinate on retirement saving and refuse, or at least postpone, to set money aside for retirement (Bernheim, Skinner, and Weinberg, 2001; Benartzi and Thaler, 2007; Choi, Laibson, and Madrian, 2011; Heimer, Myrseth, and Schoenle, 2019; Gomes et al., 2020). Only few individuals choose to annuitize savings entering retirement (Benartzi, Previtero, and Thaler, 2011) in contrast to what mainstream theory recommends (Yaari, 1965; Davidoff, Brown, and Diamond, 2005; Koijen et al., 2011; Yogo, 2016).[9] Behavioral biases translate into welfare losses of a

[7]Some papers focus on how rational investors for a given contribution rate can exploit the differential taxation of pension returns and private returns, cf., e.g., Dammon, Spatt, and Zhang (2004), Gomes, Michaelides, and Polkovnichenko (2009), Zhou (2009), and Fischer and Gallmeyer (2017). These papers do not discuss the optimal contribution rate or, more generally, how the pension system should be designed.

[8]In the 2007 U.S. Survey of Consumer Finance (SCF), 24.2% of households held stocks outside retirement accounts, a number increasing to 40.6% if retirement accounts are included (Favilukis, 2013). The median number of directly owned stocks was 3 among households in the 2001 SCF (Polkovnichenko, 2005). Other evidence from the US is reported by Blume and Friend (1975) and Goetzmann and Kumar (2008), among others, and Campbell et al. (2019) present similar findings from India, Fagereng et al. (2017) from Norway, and Florentsen, Nielsson, Raahauge, and Rangvid (2019) from Denmark.

[9]Benartzi et al. (2011) quote studies showing that only 21 percent of existing U.S. defined contribution plans offer annuities as an option, and only 6 percent of participants elects an annuity when available.

moderate or large magnitude depending on the specific setting (Calvet et al., 2007; Bhamra and Uppal, 2019). We show that biases against retirement savings, stock market participation, and annuitization induce substantial welfare losses, but also that a well-designed mandatory pension plan can reduce these welfare losses considerably by generating consumption at a higher level and with a balanced life-cycle profile.

The rest of the paper is organized as follows. Section 2 sets up the model and fixes the baseline parameter values. Section 3 presents the optimal decisions of a rational individual without a mandatory scheme, which serves as a benchmark in our welfare analysis. Section 4 explores how the presence of a mandatory defined contribution plan affects the welfare and the consumption-investment decisions of both rational individuals and individuals suffering from various behavioral biases. Section 5 identifies pension plan designs that improve the welfare for a range of individuals with both different preference parameters and different degrees of financial sophistication. Section 6 investigates the robustness of our finding to a realistic future cut in Social Security benefits. Finally, Section 7 summarizes and concludes.

2 The modeling framework

2.1 Description of the model

We use a model with annual time steps for the decision problem of an individual who has just turned t_1 years old, retires when she turns t_R years old, and may live on until the day before turning $t_M + 1$ years old. Being alive at age t, the probability of being alive at age $t + 1$ is p_t with $p_{t_M} = 0$, and $P_{t,s} = p_t \times p_{t+1} \times \cdots \times p_{s-1}$ denotes the probability of being alive at age s conditional on being alive at age t, with $P_{t,t} = 1$.

At the beginning of year t (i.e. just after turning t years old), the individual has a private, liquid wealth of F_t and a pension account balance of A_t. The individual is endowed with a liquid wealth of F_{t_1} and a pension balance of A_{t_1}. At the beginning of year t she receives income Y_t from labor, a state pension, or other sources, and

$$Y_{t+1} = Y_t R_{Yt}, \qquad R_{Yt} = \begin{cases} \exp\{\mu_{Yt} - \frac{1}{2}\sigma_{Yt}^2 + \sigma_{Yt}\varepsilon_{Yt}\} & \text{for } t = t_1, \ldots, t_R - 2, \\ \zeta & \text{for } t = t_R - 1, \\ 1 - \phi_t h - \Phi_t H & \text{for } t = t_R, \ldots, t_M - 1. \end{cases} \tag{1}$$

Here ε_{Yt} is standard normally distributed and independent over time, σ_{Yt} is the standard deviation of log income growth, and $\mu_{Yt} = \ln(\mathrm{E}_t[Y_{t+1}/Y_t])$ the expected growth rate of income. Both μ_Y and σ_Y can be age-dependent to match observed life-cycle patterns in labor income. The constant $\zeta \geq 0$ is the ratio of the annual state pension to pre-retirement income. In retirement, out-of-pocket medical costs can reduce disposable income and have a significant impact on saving and risk taking (e.g. De Nardi, French, and Jones, 2010). We let $\phi_t, \Phi_t \in \{0, 1\}$ indicate whether a health shock with a small cost h, respectively large cost H, occurs at age t, and let $q = \mathrm{Prob}(\phi_t = 1)$ denote the constant probability of a small-cost shock (e.g. for prescription medicine) and $Q_t = \mathrm{Prob}(\Phi_t = 1)$ denote the increasing-in-age probability of a large-cost shock (e.g. for nursing home spending); see the calibration below for more details.[10]

The individual pays the fraction $\alpha_t \in [0, 1)$ of pre-tax income into the pension fund and withdraws a fraction $m_t \in [0, 1)$ of the balance of the fund. We restrict α_t and m_t to depend only on age and assume that

$$\alpha_{t_R} = \alpha_{t_R+1} = \cdots = \alpha_{t_M} = 0, \quad m_{t_1} = m_{t_1+1} = \cdots = m_{t_R-1} = 0,$$

so that contributions to the pension fund are made only before retirement and withdrawals are made only in retirement. The income after pension contribution and any withdrawals from the pension fund are subject to a proportional tax given by the rate τ_Y. The disposable wealth (aka. cash-on-hand) at time t is

$$\tilde{F}_t = F_t + (1 - \tau_Y)\left[(1 - \alpha_t)Y_t + m_t A_t\right].$$

Of disposable wealth, she decides to consume a fraction $c_t \in (0, 1)$ and to invest the remainder $(1 - c_t)\tilde{F}_t$ in financial assets with a share of π_t in the stock market index

[10] For parsimony, we assume that any health-related reduction in disposable income are permanent (transitory shocks have little impact anyway), and we do not model transitions between different health states as computation time would grow proportionally with the number of states.

and the rest in the riskfree asset. The private wealth dynamics are thus

$$F_{t+1} = (1 - c_t)\tilde{F}_t R_{Ft}, \tag{2}$$

where R_{Ft} is the after-tax gross return over year t on the private investments.

We assume a constant annual log riskfree rate of r, that the log stock market return over any period dt is normally distributed with expectation $(r + \mu_S - \frac{1}{2}\sigma_S^2)\,dt$ and standard deviation $\sigma_S\sqrt{dt}$, and that returns are independent in the time dimension. The expected annual rate of return on the stock is thus $\exp\{r + \mu_S\} - 1$, i.e. μ_S captures the excess expected stock return. By assuming that the private portfolio is continuously rebalanced through year t in order to maintain a constant stock weight of π_t, the log return on the portfolio over the year is normally distributed with expectation $r + \pi_t\mu_S - \frac{1}{2}\pi_t^2\sigma_S^2$ and standard deviation $\pi_t\sigma_S$.[11] All returns on private investments—realized or not—are taxed at year-end at a proportional rate of τ_F, so that the after-tax gross return is

$$\begin{aligned} R_{Ft} &= 1 + (1 - \tau_F)\left[\exp\left\{r + \pi_t\mu_S - \frac{1}{2}\pi_t^2\sigma_S^2 + \pi_t\sigma_S\varepsilon_{St}\right\} - 1\right] \\ &= \tau_F + (1 - \tau_F)\exp\left\{r + \pi_t\mu_S - \frac{1}{2}\pi_t^2\sigma_S^2 + \pi_t\sigma_S\varepsilon_{St}\right\}, \end{aligned}$$

where ε_{St} is a standard normal random variable with contemporaneous correlation ρ_{YS} with ε_{Yt}, i.e., ρ_{YS} is the correlation between log income growth and log stock returns.

We explore different pension fund designs. One design aspect is whether it is *personal or solidary*. With a personal plan, the fund balance of an individual who passes away is distributed to the individual's beneficiaries, just as for private savings. In contrast, with a solidary plan, the balance at death is distributed to the other pension fund members which is effectively the case with typical lifelong annuities.

[11] With continuous-time notation, the stock price dynamics are $dS_t = S_t[(r + \mu_S)\,dt + \sigma_S\,dz_t]$, where z is a standard Brownian motion. If a fraction π_t of wealth is invested in the stock and the rest in the riskfree asset, the wealth dynamics are $dW_t = W_t[(r + \pi_t\mu_S)\,dt + \pi_t\sigma_S\,dz_t]$, which with a constant $\pi_t = \pi$ implies

$$W_{t+\Delta} = W_t \exp\left\{\left(r + \pi\mu_S - \frac{1}{2}\pi^2\sigma_S^2\right)\Delta + \pi\sigma_S\left(z_{t+\Delta} - z_t\right)\right\},$$

where $z_{t+\Delta} - z_t$ is normally distributed with mean 0 and variance Δ. Hence, the pre-tax rate of return over a one-year period can be written as $\exp\{r + \pi\mu_S - \frac{1}{2}\pi^2\sigma_S^2 + \pi\sigma_S\varepsilon_S\} - 1$, where $\varepsilon_S \sim N(0,1)$.

The indicator I equals 1 if the plan is solidary and 0 if personal. The pension fund invests $(1 - m_t)A_t + \alpha_t Y_t$, i.e. the time t balance plus net contribution, over year t with a share of w_t in the stock market index and the rest in the riskfree asset. As for private investments, we assume that fund portfolio is continuously rebalanced to maintain a fixed stock share of w_t through the year. The after-tax gross returns on pension investments in year t are thus

$$R_{At} = \tau_A + (1 - \tau_A) \exp\left\{ r + w_t \mu_S - \frac{1}{2} w_t^2 \sigma_S^2 + w_t \sigma_S \varepsilon_{St} \right\}, \tag{3}$$

where τ_A is the tax rate on pension returns. If the individual survives year t, next year's opening balance of the pension account is

$$A_{t+1} = [(1 - m_t)A_t + \alpha_t Y_t]\, R_{At} \frac{1}{1 - I + I p_t}. \tag{4}$$

When $I = 1$, the last term represents transfers from deceased fund members, assuming the member base is large enough that the share of realized deaths equals its expectation.

We assume that the pension payout rate is of the form

$$m_t = \left(\sum_{s=t}^{t_M} \left(1 - I + I P_{t,s}\right) \exp\left\{ - \sum_{u=t}^{s-1} \tilde{r}_u \right\} \right)^{-1}, \quad t = t_R, t_R + 1, \ldots, t_M, \tag{5}$$

where $\tilde{r}_{t_R}, \ldots, \tilde{r}_{t_M - 1}$ is referred to as the *assumed interest rate* schedule. Note that $m_{t_M} = 1$ so when the individual turns t_M years old, the fund pays out the remaining balance A_{t_M}. Also note that $m_{t+1} = e^{-\tilde{r}_t}(1 - I + I p_t) m_t / (1 - m_t)$. Clearly m_t is larger for $I = 1$ than for $I = 0$ since a larger share of the fund balance can be paid out to surviving fund members every year when the balances of deceased members are distributed to surviving members instead of external beneficiaries. The payout at $t + 1 > t_R$ is

$$m_{t+1} A_{t+1} = m_{t+1}(1 - m_t) A_t R_{At} \frac{1}{1 - I + I p_t} = m_t A_t e^{-\tilde{r}_t} R_{At},$$

so the payout is increasing [decreasing] if the realized log after-tax return $\ln R_{At}$ is greater [smaller] than the assumed interest rate $\tilde{r}_t$ at age t. We assume that

$$\tilde{r}_t = \ln \mathrm{E}_t[R_{At}] \equiv \ln\left(\tau_A + (1 - \tau_A) \exp\{r + w(t)\mu_S\}\right), \quad t = t_R, t_R + 1, \ldots, t_M - 1, \tag{6}$$

so that payouts are constant in expectation through retirement.[12] With $I = 1$, the payout stream matches that of lifelong variable annuities.[13] A fixed annuity is the case where $w_t = 0$ through retirement and $\tilde{r} = \ln(\tau_A + (1-\tau_A)(e^r - 1))$, since payouts are then constant. Also a fixed annuity comes in two forms: relatively large payouts and distribution upon death to surviving fund members ($I = 1$) or relatively small payouts and distribution upon death to external beneficiaries ($I = 0$).

The individual chooses c_t and π_t for $t = t_1, t_1 + 1, \ldots, t_M$ to maximize lifetime utility. We let J_t denote the indirect utility at time t, conditionally on being alive, and this includes the utility of consumption in year t and subsequent years, as well as any bequest utility. At the beginning of any year t, before receiving income and consuming in that year, the individual might die having a private wealth of F_t and a pension wealth of A_t. We assume this generates a bequest of $B_t = F_t + (1-I)A_t(1-\tau_Y)$ so that, if the pension balance is paid out to beneficiaries, ordinary income tax is deducted as contributions were made out of pre-tax labor income. Should the individual reach the maximum age, the pension account has already been paid out, so the bequest is then $B_{t_M+1} = F_{t_M+1}$.

We assume Epstein-Zin utility with J_t satisfying the recursive relation

$$J_t = \max_{c_t, \pi_t} \left\{ \left(c_t \tilde{F}_t \right)^{1-\frac{1}{\psi}} + \beta \, \mathrm{CE}_t^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}, \tag{7}$$

where

$$\mathrm{CE}_t = \left(p_t \mathrm{E}_t \left[J_{t+1}^{1-\gamma} \right] + (1-p_t) \mathrm{E}_t \left[\bar{U}_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \tag{8}$$

is the certainty equivalent of next period's utility which is given by J_{t+1} if the individual stays alive and the bequest utility $\bar{U}_{t+1}$ otherwise. We assume $\bar{U}_t = \xi^{\frac{1}{\psi-1}} B_t$, where $\xi \geq 0$ measures the strength of the bequest motive (see Appendix A). In addition to ξ, preferences are characterized by the relative risk aversion $\gamma > 0$, the elasticity of intertemporal substitution $\psi > 0$, and the subjective time preference factor $\beta > 0$.[14] The case $\psi = 1/\gamma$ corresponds to time-additive power utility.

[12] Expected payouts are increasing [decreasing] through retirement if $\tilde{r}_t$ is smaller [larger] than $\ln \mathrm{E}_t[R_{At}]$. We find that welfare gains are only marginally different with non-constant expected payouts than with constant expected payouts (details are available upon request). In addition, non-constant expected payouts complicate the plan design, whereas plan simplicity facilitates transparency and, probably, public support. Hence, we stick to plans with constant expected payouts.

[13] See, e.g., Charupat and Milevsky (2002) and Horneff, Maurer, Mitchell, and Stamos (2010b).

[14] We assume $\gamma \neq 1$ and $\psi \neq 1$, but cases with $\gamma = 1$ or $\psi = 1$ or both can be studied separately

Given our set-up, the indirect utility is a function $J_t = J_t(F_t, Y_t, A_t)$ of private wealth, current income, and pension wealth. We show in Appendix A that the dimension of the state space can be reduced by one by exploiting a homogeneity property; our assumptions on proportional taxation and that the state pension is proportional to pre-retirement income are needed here. More precisely,

$$J_t = (F_t + [1 - \tau_Y]A_t)\, G_t\, (y_t, a_t)\,, \tag{9}$$

where

$$y_t = \frac{[1 - \tau_Y]Y_t}{F_t + [1 - \tau_Y]A_t}, \qquad a_t = \frac{[1 - \tau_Y]A_t}{F_t + [1 - \tau_Y]A_t}, \tag{10}$$

and where G_t is linked to G_{t+1} through a recursive equation involving an expectation over the distribution of the shock ε_{St} to stock prices and the shock ε_{Yt} to labor income. The problem is solved by backwards dynamic programming on a grid of points (y_i, a_j). The expectations are approximated by Gauss-Hermite quadrature. This procedure leads to the indirect utility and the optimal decisions each year in all grid points. To obtain life-cycle patterns, we simulate many possible paths forward drawing random shocks to stock prices and labor income, using interpolation and extrapolation when simulated values of y and a are off the grid. We report averages at each age to indicate an expected life-cycle pattern. After extensive experimentation, we have settled on a 21×21 grid which produces robust and precise results with decent computation time. With our Matlab implementation on a PC with an Intel Core i7-8550U 1.8GHz processor, the calculation of the optimal private decisions and indirect utility for a given pension design takes about eight minutes. Appendix A.3 has details on the numerical solution approach.

2.2 Baseline parameter values

In subsequent examples we use the parameter values listed in Table 1. Asset market parameters are standard with a (real) riskfree rate of 1%, an equity premium of 4%, and an equity volatility of 15.7%. The assumed preference parameter values are frequently used in the theoretical life-cycle literature. With $\psi = 1/\gamma$, we take the classical time-additive power utility as the baseline setting, and assume $\gamma = 4$ which is in the range generally considered realistic based on introspection and empirical studies (Meyer and Meyer, 2005; Calvet, Campbell, Gomes, and Sodini, 2019). The

with appropriate adjustments of (7) and (8).

Parameter	Description	Value
	Financial assets	
r	Riskfree interest rate	0.01
μ_S	Expected excess stock return	0.04
σ_S	Stock volatility	0.157
	Horizon, preferences, and initial wealth	
t_1	Initial age in years	25
t_R	Retirement age in years	67
t_M	Maximum age in years	100
γ	Relative risk aversion	4
ψ	Elasticity of intertemporal substitution	0.25
β	Subjective discount factor	0.96
ξ	Bequest strength parameter	2
F_{t_1}	Initial financial wealth (kUSD)	5
A_{t_1}	Initial pension wealth (kUSD)	0
	Income	
Y_{t_1}	Initial annual income (kUSD)	40
σ_Y	Income volatility	0.1
ρ_{YS}	Income-stock correlation	0
ζ	Social Security relative to final salary	0.45
	Tax rates	
τ_Y	Income tax rate	0.30
τ_F	Tax rate on private returns	0.20
τ_A	Tax rate on pension returns	0.00

Table 1: **Baseline parameter values.**

subjective discount factor of 0.96 is a standard choice (e.g. Cocco et al., 2005), and the bequest coefficient is consistent with the empirical literature (see Kværner (2019) and the discussion therein) and values typically considered in related papers. To better match observed savings and investment decisions, several papers consider other preference parameters which may then reflect behaviorial biases rather than genuine preferences.[15] We use mortality rates from the 2017 life table for the total U.S. population (Arias and Xu, 2019).

We choose parameter values broadly matching the median 25-year old U.S. worker. The initial financial wealth is \$5,000 (no initial pension savings) in line with the

[15]For example, Fagereng et al. (2017) and Dahlquist et al. (2018) calibrate life-cycle models to observed consumption and investment patterns and find a risk aversion in the range 11-15 (to produce low stock weights) and a subjective discount factor in the range 0.75-0.93 (to produce low savings).

family net worth statistics of the 2016 Survey of Consumer Finances (SCF) and the online Net Worth Percentile Calculator of Personal Finance Data.[16] The initial annual pre-tax labor income is $40,000 and, following Cocco et al. (2005) and others, the expected labor income growth is described by a third-order polynomial; we determine the coefficients of the polynomial so that expected labor income peaks at age 55 at a value 50% above initial income and subsequently drops by 10% until retirement. The income numbers are broadly consistent with the median earnings of full-time workers in different age groups reported by the Bureau of Labor Statistics in 2020Q1.[17] We assume a 10% income volatility and a zero income-stock correlation, consistent with typical estimates in the literature (e.g. Davis and Willen, 2000; Heaton and Lucas, 2000; Cocco et al., 2005; Fagereng et al., 2017).

We fix the retirement age at 67, the current U.S. Social Security full-benefit retirement age for people born 1960 or later. The Social Security benefits depend non-proportionally on the average of the 35 career-highest inflation-adjusted annual earnings, whereas our model requires proportionality for tractability. Due to the hump-shaped income over life, the average salary over the best 35 years is typically not far from the final salary. In our baseline case, this salary level is likely to be $50-60,000, and the annual Social Security benefits in year 2020-dollars are then between 45.5% and 43.1% thereof.[18] Hence, we let $\zeta = 0.45$. The expected after-tax annual Social Security benefit is then $17,328.

Koijen, van Nieuwerburgh, and Yogo (2016, Table III) report out-of-pocket health expenses in year 2005-dollars at selected ages from 51 to 93. Our model includes medical costs in retirement, so we consider the mean expenses at ages 72, 79, 86, and 93 in excess of expenses at age 65, and adjust for inflation until 2020.[19] Relative to the expected after-tax Social Security benefits, the mean excess expenses constitute 2.3% at age 72, 10.9% at age 79, 28.9% at age 86, and 181.9% at age 93. A similar exercise based on De Nardi, French, Jones, and McCauley (2016, Figure 3)

[16] See SCF Table 2 in Bricker et al. (2017) and https://personalfinancedata.com/networth-percentile-calculator/, accessed April 28, 2020.

[17] The median annual earnings are $31,460 (20-24yrs), $45,344 (25-34yrs), $56,160 (35-44yrs), $57,252 (45-54yrs), $56,264 (55-64yrs), and $48,776 (65+ yrs), respectively. Source: https://www.bls.gov/news.release/wkyeng.t03.htm, accessed April 28, 2020.

[18] Using 2020 as the year of eligibility, the monthly benefits are $0.9x + 0.32\max\{0, \min\{x - 960, 4825\}\} + 0.15\max\{0, x - 5785\}$, where x is the monthly salary. Source: https://www.ssa.gov/oact/cola/piaformula.html, accessed on April 28, 2020.

[19] We multiply by the ratio $257.971/190.7 \approx 1.353$ of CPI in January 2020 to January 2005. Source: https://www.usinflationcalculator.com/inflation/consumer-price-index-and-annual-percent-changes-from-1913-to-2008/, accessed on May 20, 2020.

leads to average out-of-pocket expenses of 3.2%, 9.9%, 20.0%, and 41.7% at the same ages, i.e., significantly lower late-life medical expenses. We assume that every year in retirement there is a probability $q = 15\%$ of a shock reducing income by $h = 3\%$ and a probability of $Q_t = \min\{0.03 \times \frac{t-t_R}{t_M-t_R} + \left(\frac{(t-t_R-15)^+}{t_M-t_R-15}\right)^2, 0.5\}$ of a shock reducing income by $H = 85\%$. The probability of a large shock grows linearly until 15 years into retirement where it starts growing faster until it reaches 50%. Based on simulations of our model, medical costs are expected to be 2.9%, 10.4%, 26.6%, and 85.3% of Social Security pay at ages 72, 79, 86, and 93, respectively, giving a good match with the empirical estimates with the late-in-life expenses falling in between the estimates from the two papers. Our two-shock structure also produces a large dispersion in medical expenses across individuals, as reported by De Nardi et al. (2016). Even a small risk of large expenses can affect saving and portfolio decisions.

Finally, we assume a 30% tax on income and 20% on private returns—in the range of tax rates across U.S. states—and no tax on returns on pension savings. Contributions to the pension scheme are tax-free, whereas payouts are subject to income tax.

2.3 Welfare metric

In our setting, the pension fund design is characterized by a sequence of contribution rates $\alpha_{t_1}, \alpha_{t_1+1}, \ldots, \alpha_{t_R-1}$ and stock weights $w_{t_1}, w_{t_1+1}, \ldots, w_{t_M-1}$, as well as the annuitization indicator I. For simplicity, we represent such a design by (α, w, I).

A key element of our analysis is to compare the utility or welfare that an individual can generate under different designs of the mandatory pension plan but under the same assumptions about income, initial wealth, etc. Suppose we want to compare two pension designs, (α, w, I) and $(\hat{\alpha}, \hat{w}, \hat{I})$. The individual obtains an initial indirect utility of $J = J_{t_1}(F, Y, A; \alpha, w, I)$ with the first pension design and $\hat{J} = J_{t_1}(F, Y, A; \hat{\alpha}, \hat{w}, \hat{I})$ with the second design. If $J > \hat{J}$, we can quantify how much better off the individual is with (α, w, I) than with $(\hat{\alpha}, \hat{w}, \hat{I})$ by the fraction λ of additional life-time labor income and initial wealth that the individual would need to receive under the design $(\hat{\alpha}, \hat{w}, \hat{I})$ to obtain the same lifetime utility as with the design (α, w, I) and the actual income and wealth. With the additional income and wealth, the indirect utility with the design $(\hat{\alpha}, \hat{w}, \hat{I})$ is

$$J_{t_1}\left([1+\lambda]F, [1+\lambda]Y, [1+\lambda]A; \hat{\alpha}, \hat{w}, \hat{I}\right) = (1+\lambda)J_{t_1}\left(F, Y, A; \hat{\alpha}, \hat{w}, \hat{I}\right). \tag{11}$$

Equating this with $J_{t_1}(F, Y, A; \alpha, w, I)$, we find

$$\lambda = \frac{J_{t_1}(F, Y, A; \alpha, w, I)}{J_{t_1}(F, Y, A; \hat{\alpha}, \hat{w}, \hat{I})} - 1. \tag{12}$$

For $\hat{\alpha} = 0$ (then $\hat{w}$ and $\hat{I}$ are meaningless), the denominator corresponds to the case without mandatory savings, and λ then measures the welfare gain from imposing the plan (α, w, I) relative to the situation without a mandatory scheme.

We can transform the relative utility gain λ into an initial dollar amount if we multiply λ by the sum of the initial financial wealth and the present value of after-tax income from labor until retirement and Social Security less out-of-pocket medical expenses in retirement. Following Hall (1978), Guiso and Sodini (2013), and others, we calculate this present value by discounting expected after-tax labor income by the riskfree rate of 1%. Given the assumptions about labor income and relevant parameter values, the present value of income turns out to be \$1,468,900 (based on 10,000 simulated income paths), which is substantially larger than the assumed initial financial wealth of \$5,000.[20] In this case, a welfare gain λ of 1% corresponds to \$14,739.

3 Benchmark: rational individual without mandatory plan

This section revisits the case of a rational individual without a mandatory pension plan as discussed in the existing literature on individuals' optimal life-cycle consumption and investment decisions, cf., e.g., Viceira (2001) and Cocco et al. (2005). Our model is relatively rich by including taxes on income and returns, Social Security retirement benefits, and out-of-pocket medical costs. This version of our model serves as a benchmark when we introduce a mandatory pension scheme as such a scheme intuitively should bring an irrational individual closer to the ideal life-cycle consumption pattern.

[20] Since the income is not fully spanned by traded assets, there is no unique way to calculate the present value of future income.

3.1 No annuitization

First, we disregard any annuitization of retirement savings. Figure 1 illustrates the key properties of the model, all complying with the well-known results in the life-cycle literature. The solid, smooth blue curve in Panel A is the expected annual income after taxes and out-of-pocket medical expenses with the dashed lines indicating the 10th and 90th percentiles based on 10,000 simulated paths. The orange curves illustrate the optimal annual consumption over the life cycle with the thin solid lines depicting three paths. The solid, smooth orange curve confirms that expected consumption is hump-shaped over the life cycle in line with observed average consumption in the data (Thurow, 1969; Gourinchas and Parker, 2002). Note that, in spite of the individual's consumption smoothing preference, optimal consumption remains quite volatile as reflected by both the depicted percentiles and consumption paths.

Part of pre-retirement income is saved and invested in financial assets in order to finance consumption exceeding disposable income in retirement. The amount saved in year t is after-tax income less consumption, $(1 - \tau_Y)Y_t - c_t\tilde{F}_t$, and we define the saving rate in year t as the amount saved relative to after-tax income. Note that, by drawing on accumulated savings, consumption may exceed after-tax income in a given year and thus generate a negative saving rate. Panel B of Figure 1 shows that the mean saving rate starts out at around 20%, declines slowly until age 45-50 and then more rapidly to reach negative numbers from age 60. In addition to the individual's age, the optimal saving rate depends on the state, i.e. the individual's income and accumulated wealth. The percentiles and paths depicted illustrate the widening of the saving rate distribution as the time horizon increases.

Panel C shows that financial wealth is accumulated until retirement, after which the rational individual gradually draws down wealth to finance consumption, although rather slowly due to both possible health shocks and the bequest motive. The wide range of wealth levels indicated by the 10th and 90th percentile curves reflect uncertainty both about future income and returns on investments. Finally, Panel D illustrates that the stock weight—i.e. the fraction of financial wealth optimally invested in the stock market—is 100% until the 50s and then declines almost linearly with age. The glidepath pattern emerges because of the gradual decline in human capital relative to financial wealth. The decline in the stock weight in retirement is induced primarily by the risk of health shocks reducing disposable income

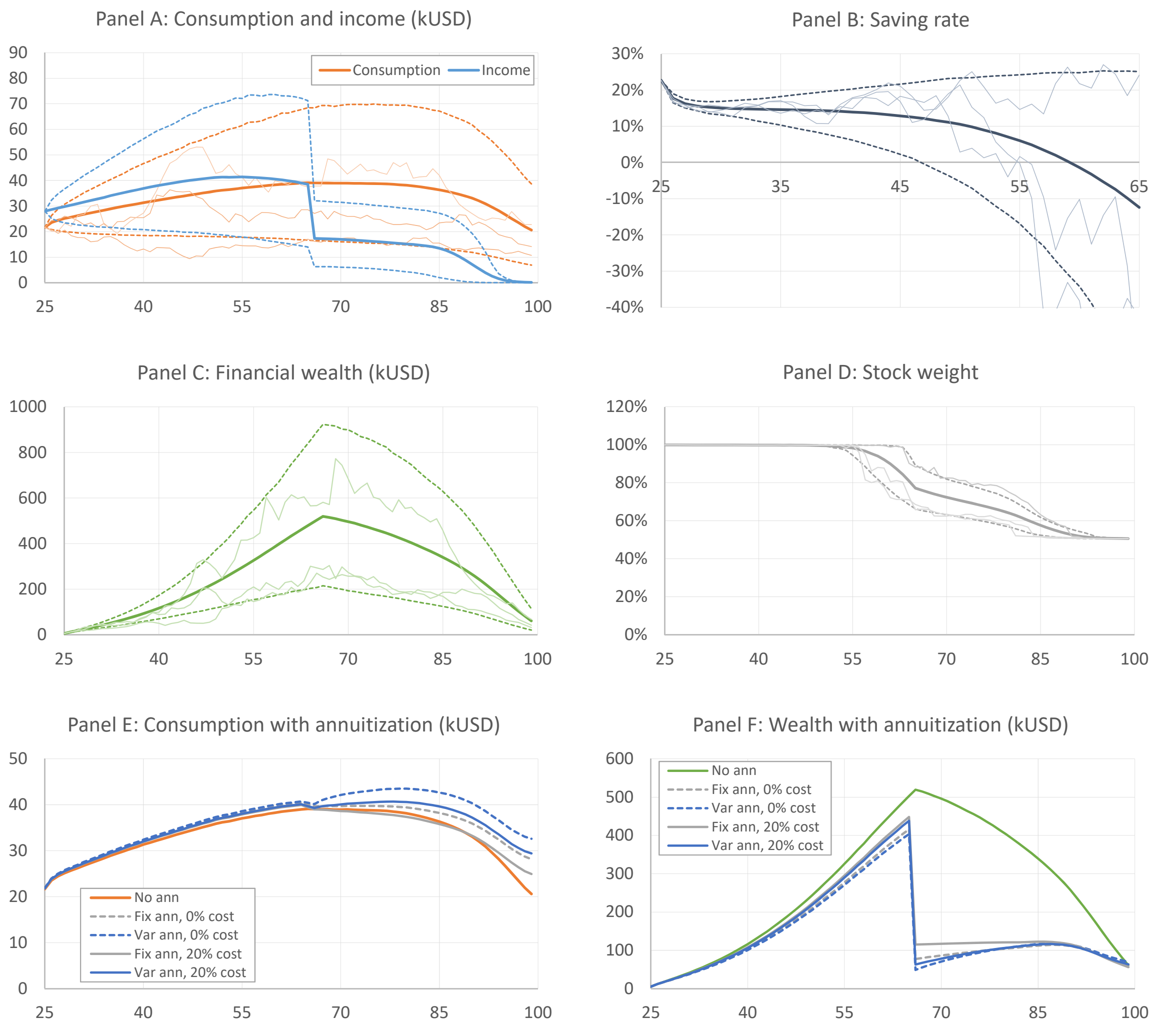

Figure 1: **The life of a rational individual without pension.** The individual's age is depicted along the horizontal axis in all panels. Panel A shows the annual consumption (orange lines) and after-tax labor income (blue lines), Panel B the saving rate, Panel C the accumulated financial wealth, and Panel D the fraction of wealth invested in the stock market. In Panels A-D, the thick solid line represents the mean across 10,000 simulated paths, whereas the dashed lines indicate the 10th and 90th percentile at each age. The thin solid curves represent three paths. Panel E shows mean consumption and Panel F mean financial wealth in the case without the option to annuitize as well as in cases with either a lifelong fixed or 50/50 variable annuity with either zero or 20% costs. The baseline parameter values listed in Table 1 are used.

below Social Security benefits.

3.2 Annuitization

The above analysis ignores annuitization of retirement wealth. The individual must have large savings at retirement and consume fairly little through retirement in order to uphold a decent consumption level in the case that she lives long, but if she then dies early in retirement she would leave a suboptimally large bequest. If she ignores the possibility of living long, she runs the risk of "outliving her savings" and end up consuming very little in her final years. Theoretically, annuities strike the right balance by offering a retirement income until death and thus insurance against outliving your savings. Upon death, the remaining annuitized wealth of the deceased is retained by the annuity issuer to cover payments to the issuer's longer-lived annuity holders as well as costs and profits, i.e. a well-functioning annuity market allows individuals to share longevity risk. Appendix A.2 extends our model with an option to annuitize.

We consider both (i) a fixed, lifelong annuity where the annuitized savings are invested in the riskfree asset and (ii) a variable, lifelong annuity where savings are invested in a portfolio of the riskfree and the risky asset with constant weights. First, assume fairly priced annuities so that annuity payments are based on the entire population's mortality rate and the annuity issuer makes a zero expected profit. With access to a fixed annuity, the individual obtains a 4.0% larger lifetime utility than in the baseline case without annuitization; on average she annuitizes 81.2% of her savings at retirement. With a variable annuity corresponding to 50% stocks and 50% bonds, the utility gain is 4.6% and she annuitizes, on average, 87.4% of savings at retirement. Next, assume a 20% annuity cost so that payouts are 20% lower than for an actuarially fair annuity. With a fixed annuity, the average annuitization then drops to 74.7% of retirement savings and the welfare gain to 3.0%. With the 50/50 variable annuity, the average annuitization is now 85.2% and the welfare gain 2.5%.[21] Panels E and F in Figure 1 show that the option to annuitize lead to larger consumption late in life and lower wealth accumulation up to retirement. Despite a significant cost rate, annuitization remains beneficial to the individual with the

[21] Among variable annuities with a constant stock share divisible by 5%, a 50% share is optimal with 20% costs. With zero costs, both a 50% and a 55% stock share produce a 4.6% welfare gain.

baseline parameter values.[22] Nevertheless, few people choose to annuitize wealth at retirement, maybe due to product non-transparency, limited financial literacy, adverse selection issues, or even higher costs in current annuity markets. With only few people annuitizing, the issuers have a hard time estimating the mortality risks of customers and to rely on diversification of mortality risk across customers, which can justify high costs.

4 Welfare implications and financial sophistication

This section explores how mandatory pension schemes affect individuals with varying degrees of financial sophistication. As explained in the Introduction, a large body of research has documented that many individual investors fail to follow the theoretically optimal consumption and investment strategy. We focus on two behavioral biases: non-participation in stock markets and procrastination on retirement saving. Clearly, mandatory participation in a pension plan with a non-trivial contribution rate and significant stock holdings may improve the welfare of such "irrational" individuals. Our setting allows us to quantify the welfare gains generated by any specific pension plan design and search for the optimal design. We also discuss how mandatory schemes affect the welfare and private decisions of a rational individual. Throughout this section, we apply the baseline parameter values listed in Table 1. In particular, we fix the relative risk aversion to 4 and the elasticity of intertemporal substitution to 1/4. In the next section, we vary these parameters and search for the best pension design for a group of heterogeneous individuals.

In our setting, a mandatory pension plan is defined by sequences of contribution rates $\alpha_{t_1}, \alpha_{t_1+1}, \dots, \alpha_{t_R-1}$ and pension fund stock weights $w_{t_1}, w_{t_1+1}, \dots, w_{t_M-1}$, and the personal/solidary payout indicator $I \in \{0, 1\}$. To structure and streamline the analysis, we focus on relatively simple plan designs; simplicity is also important for transparency and thus public support of a broad or even universal mandatory retirement saving program. More specifically, we assume a constant contribution rate α either applying immediately from age 25 or from a later age and always

[22] For a given cost rate, rational individuals with low risk aversion and low elasticity of intertemporal substitution annuitize a lower fraction of retirement savings. With a sufficiently large cost rate, rational individuals prefer not to annuitize at all.

until retirement. We focus on the following set of fund investment policies each characterized by how the stock weight varies with age:

IP1: 50% stocks all life,

IP2: stock weight at age t is $(120 - t)\%$,

IP3: stock weight at age t is $\min\{150 - t, 100\}\%$,

IP4: stock weight is 100% until age 52, then slopes to 40% at retirement and stays there,

IP5: stock weight is 100% until age 52, then slopes to 60% at retirement and stays there,

IP6: stock weight is 100% until age 57, then slopes to 80% at retirement and stays there,

IP7: 100% stocks all life.

The first and last policies feature a constant stock weight of 50% or 100% through life, whereas the other policies have a glidepath element with a declining stock weight over some age interval. IP2 is the '120-minus-age' strategy seemingly popular among some financial advisors, and IP3 is the similar '150-minus-age' strategy with a cap at 100%. The policies IP4-IP6 feature a stock weight starting at 100% and then declining over the last 10 or 15 years before retirement to reach a constant level through retirement at 40%, 60%, or 80%. Finally, given the welfare benefits of lifelong annuities reported in Section 3.2, we assume that the plan is solidary ($I = 1$). For each pension design we calculate a utility gain by using (12) with the denominator derived from the case without a mandatory pension plan and without private annuitization.

4.1 Effects on individual welfare

First, we consider individuals not privately participating in the stock market. In the absence of a mandatory saving plan (as well as private annuitization options), the lifetime utility of a stock avoider is 9.9% lower than when the individual invests optimally in both stocks and bonds. The lack of stocks in her portfolio reduces the expected returns and thus her expected future wealth and consumption possibilities. For example, her expected wealth at retirement is more than 20% lower and her expected consumption at age 80 is 31% lower than with the optimal life-cycle stock-bond allocation.

Panel A of Table 2 shows percentage welfare gains of the stock avoider for pension plans demanding 6%, 10%, 14%, or 18% contributions from age 25, 30, 35, or 40, and for the seven investment policies outlined above. All the designs lead to sizeable welfare gains relative to the stock avoider's lifetime utility without a mandatory saving plan. As expected, a later contribution initiation age implies a larger preferred contribution rate. Note that the investment policy has a smaller welfare impact than the contribution rate and the contribution initiation age. Among the plans covered by the table, the maximum welfare gain is 11.6%—about $171,000 in present value terms—and is achieved with 10% contributions beginning from age 30 and the investment policy IP5, but many plan designs lead to welfare gains above 10%. Looking through a wider range of pension plans, we find a slightly larger gain of 11.8% with 9% contributions from age 30 and investment policy IP5. The best pension plan design brings the utility of the stock avoider roughly to the level of the rational individual without a pension plan and without an annuitization option. Individuals generally dislike significant mandatory savings into an illiquid pension account early in life since they want to build a liquid wealth buffer they can draw upon in case of negative income shocks. On the other hand, for the stock avoider, early contributions to the pension fund is the only way to obtain a position in the stock market, which improves utility.

Next, we consider the behavioral bias to procrastinate on retirement saving. For convenience, we model procrastination as follows. The utility the individual derives from any consumption plan is associated with the baseline subjective discount factor $\beta = 0.96$. However, due to lack of self-control, the individual applies a lower value of β (i.e., is more impatient) when making decisions, so that she consumes too much early in life and builds up too little wealth.[23] To isolate the effects of procrastination, we assume that the individual is able to optimally invest in the stock index.

We assume the procrastinator makes decisions using $\beta = 0.85$. The expected wealth just before retirement is then 185 kUSD (i.e., thousand USD) or only about 35% of the 520 kUSD when decisions are made with the correct $\beta = 0.96$ (without private annuitization). The lower wealth accumulation fits well with the 224.1 kUSD median net worth of U.S. families in which the head of family is of age 65-74 years (see Bricker et al., 2017, Table 2) as our model is for an individual saver instead of a family. Based on Norwegian data, Fagereng et al. (2017) estimate the subjective

[23]Procrastination can also be modeled using (quasi-)hyperbolic discounting (Laibson, 1997).

	Age 25-66				Age 30-66				Age 35-66				Age 40-66			
	6%	10%	14%	18%	6%	10%	14%	18%	6%	10%	14%	18%	6%	10%	14%	18%
Panel A: Stock avoider																
IP1	10.0	9.9	7.1	3.0	8.9	10.7	9.9	7.7	6.9	9.8	10.7	9.9	5.1	7.7	9.5	10.2
IP2	11.0	10.4	7.3	3.1	9.9	11.3	10.2	7.9	7.7	10.5	11.1	10.2	5.6	8.3	10.1	10.7
IP3	11.4	10.4	7.1	3.0	10.6	11.5	10.1	7.7	8.6	11.0	11.3	10.2	6.4	9.0	10.4	10.8
IP4	11.3	10.4	7.2	3.0	10.4	11.5	10.2	7.8	8.3	10.9	11.3	10.2	6.0	8.7	10.4	10.8
IP5	11.5	10.5	7.2	3.0	10.6	11.6	10.2	7.8	8.5	11.0	11.4	10.3	6.2	9.0	10.5	10.9
IP6	11.3	10.3	7.1	2.9	10.5	11.4	10.1	7.7	8.6	10.9	11.2	10.1	6.4	8.9	10.3	10.7
IP7	11.0	10.0	6.9	2.8	10.2	11.1	9.8	7.5	8.4	10.5	10.8	9.8	6.3	8.7	10.0	10.3
Panel B: Procrastinator																
IP1	40.3	42.3	39.1	34.2	36.7	42.9	42.5	39.5	30.2	39.9	42.3	41.4	21.8	33.7	39.2	40.8
IP2	42.0	43.1	39.6	34.5	38.7	43.9	43.0	39.8	32.4	41.2	43.1	41.8	23.9	35.2	40.2	41.4
IP3	40.9	42.4	39.2	34.2	37.5	43.0	42.4	39.4	31.2	40.2	42.4	41.3	22.7	34.1	39.3	40.8
IP4	42.2	43.1	39.6	34.5	39.1	44.0	43.1	39.8	33.1	41.5	43.2	41.9	24.8	35.8	40.5	41.6
IP5	41.6	42.9	39.5	34.4	38.4	43.6	42.8	39.7	32.1	40.9	42.9	41.7	23.6	35.0	40.0	41.3
IP6	40.3	42.1	39.0	34.1	36.8	42.6	42.2	39.2	30.4	39.6	42.0	41.0	21.9	33.3	38.7	40.4
IP7	38.6	41.0	38.3	33.6	34.9	41.1	41.2	38.5	28.3	37.8	40.6	40.0	19.9	31.2	36.9	38.9
Panel C: Rational																
IP1	3.0	2.5	-0.1	-3.8	2.8	3.8	3.2	1.4	2.2	3.5	4.1	3.7	1.7	2.7	3.6	4.1
IP2	3.7	2.9	0.1	-3.6	3.4	4.3	3.5	1.5	2.5	3.9	4.4	3.9	1.8	3.0	3.9	4.4
IP3	3.6	2.8	0.0	-3.7	3.3	4.2	3.4	1.4	2.5	3.8	4.3	3.8	1.7	2.8	3.7	4.2
IP4	3.9	3.0	0.2	-3.6	3.6	4.5	3.6	1.6	2.7	4.1	4.6	4.0	1.9	3.1	4.0	4.5
IP5	3.8	2.9	0.1	-3.6	3.4	4.4	3.5	1.5	2.6	3.9	4.4	3.9	1.8	2.9	3.9	4.4
IP6	3.5	2.7	0.0	-3.7	3.2	4.1	3.3	1.4	2.4	3.7	4.2	3.7	1.7	2.7	3.6	4.1
IP7	3.2	2.4	-0.2	-3.9	3.0	3.8	3.0	1.1	2.2	3.3	3.8	3.3	1.5	2.5	3.3	3.7

Table 2: **Percentage utility gain for selected pension designs.** The utility gain is in percent of the utility in the no-pension case without private annuitization. Panel A is for an individual not privately holding stocks, Panel B is for an individual procrastinating on savings (by taking decisions using $\beta = 0.85$), and Panel C is for a rational individual. The baseline parameter values listed in Table 1 are used, in particular the relative risk aversion is $\gamma = 4$ and the elasticity of intertemporal substitution is $\psi = 1/4$. The pension plans have the solidary payout feature ($I = 1$) and constant expected payouts. For each investor type and contribution period, the maximum gain is written in red color.

discount factor to be even lower, $\beta = 0.80$, in order to match the limited savings of Norwegian households. To be clear, we assume that $c_t(a_t, y_t)$ and $\pi_t(a_t, y_t)$ are derived by the dynamic programming technique assuming $\beta = 0.85$ and then evaluated with $\beta = 0.96$. The evaluation of a given strategy (c_t, π_t) follows the same backwards iterative approach as when deriving the optimal strategy, except that no maximization is performed.

As expected, the procrastinator benefits significantly from a mandatory pension plan. Panel B of Table 2 shows the welfare gains for the procrastinator for selected pension plan designs. All of the gains are substantial and many exceed 40%. Among the designs covered by the table, the largest gain of 44.0%—or around $648,500 in present value terms—is achieved with 10% contributions from age 30 and investment policy IP4. A marginally larger gain of 44.1% is obtained with 11% contributions from age 30.

Finally, we now turn to the rational individual. A mandatory plan restricts the individual's decisions, but brings tax advantages and an access to fair annuitization. Panel C of Table 2 shows that the maximum welfare gain is 4.6%—corresponding to around $67,800 in present value terms—for enrollment at age 35 with a 14% contribution rate and investment policy IP4; no non-reported combination of contribution rate and initiation age gives a larger gain. The welfare gain is at par with the gain the individual could obtain in the no-pension setting with the option to privately annuitize her desired fraction of retirement savings at zero cost, and clearly exceeds the 2.5% gain with a 20% cost on private annuitization. The welfare gains are relatively insensitive to the investment policy since the rational individual to a large extent can undo undesired pension fund investments by adjusting private investments. Note that a plan with relatively large contributions from age 25 leads to welfare losses due to the excessive savings in an illiquid account.

When looking across the three panels in Table 2, we notice that the pension plans generating large gains are similar across investor characteristics. For all three types, some of the largest gains are for plans with a moderately aggressive glidepath investment policy and either 10% contributions from age 30 or 14% contributions from age 35. Section 5 expands this analysis by considering a range of investor preferences and concludes that a solidary plan with 9% contributions from age 30 and investment policy IP4 leads to the largest average welfare gain among the plans that generate welfare gains for all investor types considered. In the following subsection we investigate how this plan affects the life-cycle decisions of participants.

4.2 Effects on life-cycle decisions

In order to better understand how a mandatory pension plan affects the welfare of individuals, Figure 2 investigates how the presence of a mandatory plan changes an individual's consumption and investment decisions over the life cycle. The plan we consider has 9% contributions from age 30, investment policy IP4, and solidary payouts that are constant in expectations.

Individual welfare in our model is determined by the consumption at different ages and in different states. Panel A shows expected consumption each year for different individuals. The dashed blue curve is for an individual not participating in a mandatory pension plan but who can choose the fraction of retirement wealth to annuitize via a lifelong variable annuity corresponding to equal stock and bond investments. All other curves assume no private annuitization. The two grey curves are for a rational individual with the dotted curve representing the case without a mandatory plan and the solid curve the case with a mandatory plan. Similarly, there are two orange curves for the stock avoider and two green curves for the procrastinator. The mandatory plan reduces consumption before retirement a lot for the procrastinator and more marginally for the rational individual and the stock avoider. In contrast, with a mandatory plan, all three types of individuals consume a lot more in retirement, especially late in retirement. In retirement, all three consume even more than the rational annuitizer, which at first may seem surprising. The reduced return taxation in the mandatory plan compared to private annuitization facilitates larger lifetime consumption, but a rational individual should want to smooth that out over the life cycle. Of course, a very large contribution rate would naturally lead to a pattern with low consumption before and high consumption in retirement. But with the pension plan considered here, the rational individual with a mandatory pension plan still builds up substantial private savings, although she could choose to save less outside the pension plan, which would lead to a smoother consumption profile closer to the profile of the rational annuitizer. However, welfare is not determined solely by expected consumption, and the individual prefers to keep a liquid wealth buffer for ensuring a decent consumption even if bad states (low income, low returns) are realized.

Panel B depicts expected wealth of the life cycle. The dotted curves show private

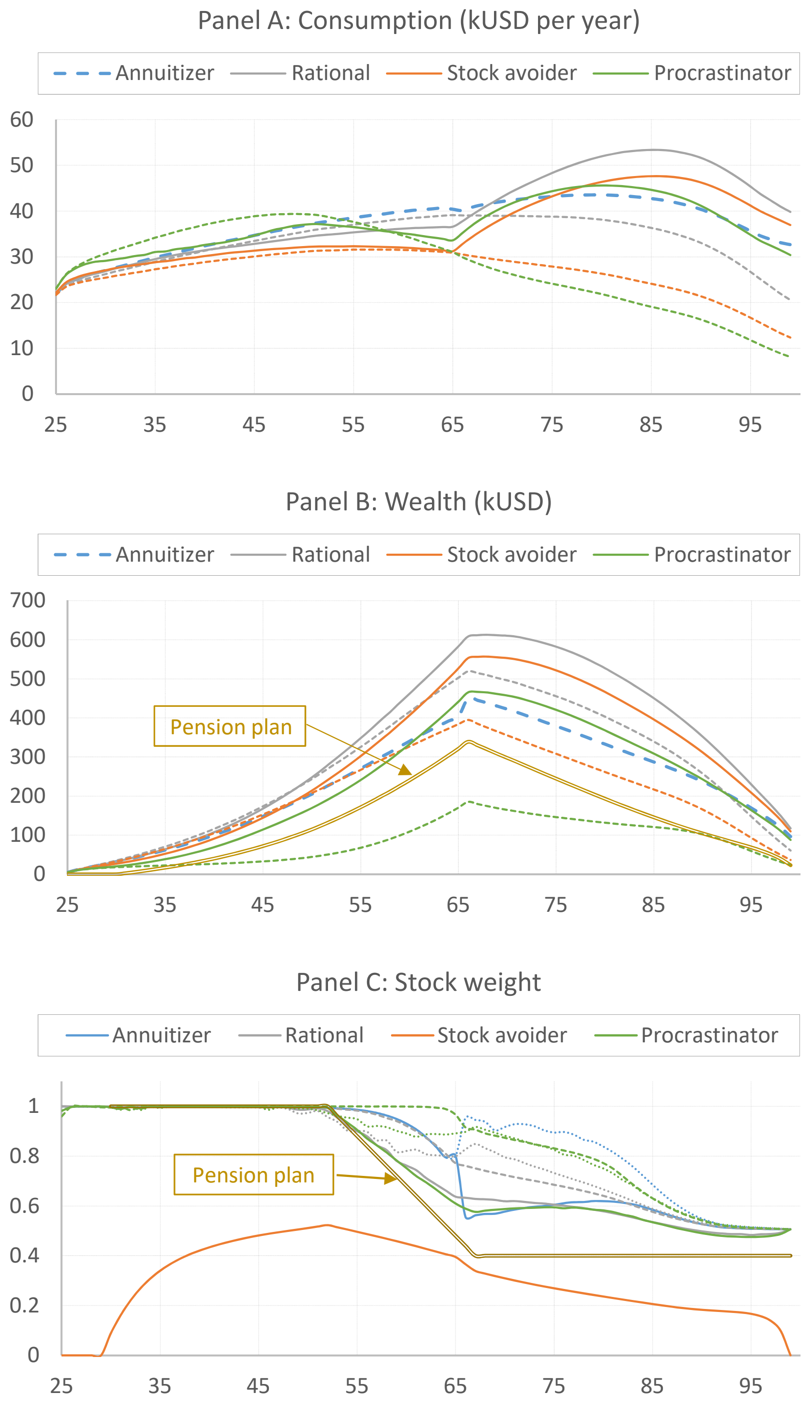

Figure 2: **Consumption, wealth, and portfolio with or without a pension plan.** Age is shown along the horizontal axis. Blue lines are for an individual without a mandatory pension plan, but with an optimal fraction of wealth at retirement invested in a lifelong variable annuity with 50% stocks and 50% bonds. The other colors refer to the case without private annuitization and either without (dashed lines) or with (solid lines) a pension plan having 9% contributions from age 30, investment policy IP4, and solidary payouts that are constant in expectations. In Panel B, the solid curves show total wealth, i.e., the sum of private wealth F_t and after-tax pension wealth $(1 - \tau_Y)A_t$. In Panel C, the dotted lines show the fraction of private wealth invested in stocks, whereas the solid lines represent the fraction of total wealth invested in stocks both privately and within the pension plan (or annuity plan). All lines are based on means across 10,000 simulated paths. The baseline parameter values listed in Table 1 are used.

wealth in the case without a mandatory savings scheme. The solid curves show total wealth (private wealth F_t plus after-tax pension wealth $(1 - \tau_Y)A_t$) in the case with mandatory savings. The dashed blue curve for the annuitizer shows private wealth plus the value of the chosen annuity. In all cases, wealth peaks around retirement and then decumulates through retirement, at first slowly due to the risk of future medical expenses. The after-tax value of pension savings at retirement is expected to be around 339 kUSD with the specified plan. Besides the savings in the pension fund, the individuals maintain substantial liquid private savings, and their total savings clearly exceed what they would save without the mandatory plan. At retirement, expected total savings are 465 kUSD for the procrastinator (339 kUSD in the pension fund and 126 kUSD in private savings) compared to only 185 kUSD in the absence of the plan. The fact that individuals prefer to have sizeable liquid savings to buffer against bad times implies that a mandatory savings plan alone should not generate as large retirement wealth as the rational individual would accumulate without a mandatory scheme.

Panel C illustrates the asset allocation over life. The dotted curves show the fraction of private wealth invested in stocks with the small-dot curve representing the case with a mandatory pension plan and the wide-dot curve the case without a mandatory plan. The solid curves represent the fraction of total wealth invested in stocks both privately and within the pension plan (or annuity plan). By definition, the stock avoider has no private stock investments, but obtains a sizeable stock market exposure when participating in the pension plan (orange curve), although this exposure still falls short of the ideal. The other individuals invest almost all of their private wealth in stocks until around age 50 whether or not they participate in the mandatory pension plan that also is fully invested in stocks in this life phase. Both the stock avoider and the rational individual keep a large average fraction of their private wealth in stocks until late in retirement, which brings their total stock weight above the stock weight in the pension plan. However, if they are hit by large medical cost shocks or just very poor returns, they reduce their private stock investments considerably, while the asset allocation mix of the pension savings remains the same.

4.3 Sources and robustness of welfare gains

The above analysis has illustrated the impact of a mandatory pension scheme on individuals' welfare and life-cycle consumption and investment decisions. Still assuming RRA = $(\text{EIS})^{-1} = 4$, welfare improves by 11.7% for the stock avoider, 43.6% for the procrastinator, and 4.5% for the rational individual when they are forced to save 9% of income from age 30 in a pension fund following investment policy IP4 and featuring life-long, solidary, and constant expected payouts. Table 3 shows how these welfare gains are impacted by variations in some assumption and plan features.

First, the welfare gain is partly due to returns inside the pension fund being tax-exempt. However, if pension returns were subject to the same 20% taxation as private returns, the welfare gain is only marginally reduced to 10.9% for the stock avoider, 42.3% for the procrastinator, and 3.8% for the rational individual. Although not being a major source of welfare gain for these individuals, the tax advantage to pension savings may be crucial for making more risk averse and intertemporally inelastic rational individuals willing to participate in a broad or universal mandatory pension scheme, cf. our discussion in Section 5 below. By hedging against the "risk" of living long, the solidary payout feature is a substantial driver of the welfare gains from the pension plan. Removing the feature, the welfare gain drops significantly to 7.5% for the stock avoider, 29.5% for the procrastinator, and only 0.7% for the rational individual. When removing both the tax advantage and the solidary feature, the welfare of the rational individual is reduced when being forced into the pension plan. Note, however, that the welfare loss is only 0.1%, which reflects the rational individual's ability to almost completely undo any undesired implications of the pension plan by adjusting her private saving and asset allocation decisions. Even without the tax advantage and the auto-annuitization of a solidary plan, both the stock avoider and the procrastinator gain from participating in the pension plan.

Due to economies of scale, access to more asset classes, or simply better information and skills, pension fund managers may be better investors than private investors, even after accounting for management fees. Our baseline assumption is that both the private investor and the fund manager face a riskfree rate of 1% and a risky asset portfolio with an expected excess return of 4% and a volatility of 15.7%. Table 3 shows that if the fund manager can up the expected excess return to 4.5% for the same volatility, the welfare gains increase to 12.8%, 45.2%, and 5.3% for the three investor types. Conversely, if the fund manager can obtain the expected excess

	Stock avoider	Procrastinator	Rational
Baseline plan	11.7	43.6	4.5
No tax advantage ($\tau_A = 0.2$)	10.9	42.3	3.8
Personal plan ($I = 0$)	7.5	29.5	0.7
No tax adv. ($\tau_A = 0.2$), personal plan ($I = 0$)	5.5	26.6	-0.1
Larger exp. return in fund (+0.5 pct points)	12.8	45.2	5.3
Lower volatility in fund (-1 pct point)	12.3	44.4	4.8
Medical costs (h, H) reduced 1/3	12.3	44.8	4.9
No medical costs	13.4	46.6	5.9

Table 3: **Robustness of welfare gains.** The utility gain is in percent of the utility in the no-pension case without private annuitization. The baseline plan has 9% contributions from age 30, investment policy IP4, solidary payouts ($I = 1$) with constant expected values.

return of 4% with a volatility of only 14.7%, the gains are 12.3%, 44.4%, and 4.8%, respectively.

Our model includes the risk of substantial late-life out-of-pocket medical expenses—a key concern of US households. Such costs affect life-time utility both with and without a mandatory pension scheme. Since medical costs cannot be paid out of illiquid pension savings, a mandatory pension scheme is less appreciated when medical costs are large, but Table 3 shows that gains are relatively insensitive to the assumed costs.

5 Pension plan designs for a range of individuals

A typical mandatory pension plan covers a large number of individuals with different preferences and degrees of financial sophistication. The question then is how to specify the contribution, investment, and payout policies to serve a set of heterogeneous participants in the best possible manner. We assume the plan covers both rational individuals, stock avoiders, and procrastinators. Within each of these groups, we consider nine set of preferences generated by combining an RRA (γ) of 2, 4, or 6 and an EIS (ψ) of 1/6, 1/4, and 1/2, leading to a total of 27 subgroups of participants. The RRA-values span the interval generally considered realistic based on introspection and empirical studies (Meyer and Meyer, 2005; Calvet et al., 2019). Recall that with EIS = 1/RRA, Epstein-Zin recursive utility is equivalent to the time-additive expected CRRA utility traditionally assumed in financial economics, and in this case the individual is indifferent to the timing of the resolution of uncertainty. With EIS $< 1/\text{RRA}$ [resp. EIS $> 1/\text{RRA}$], the individual prefers late [early] resolution of uncertainty. The values of EIS and RRA we consider thus cover cases with different timing preferences.[24]

For each pension plan design considered we calculate the welfare gain for each of the 27 participant types. We identify designs that improve the welfare of all 27 types as this seems desirable for obtaining broad public support for the introduction of a mandatory plan. Among those designs we maximize the average welfare gain across the 27 types. More specifically, for a given contribution initiation age, we look for the (integer) contribution rate that leads to the largest average welfare gain conditional on the welfare gain being positive for all 27 participants. We do this for a contribution initiation age of 25, 30, 35, and 40; it will be clear that initiation later than at age 40 reduces the average welfare gain. The average welfare gain criterion can be interpreted as applying a specific social welfare function giving equal weights to the 27 participant types. Solid knowledge of the distribution of types across preferences and financial sophistication is unavailable, but the empirical behavioral finance literature suggests that more than one third of the adult population are procrastinating on retirement savings. As procrastinators gain the most from mandatory pension plans, the simple average gain we report is probably conservative. Furthermore, some procrastinators may also avoid stocks and would thus gain even more from the mandatory plans we consider. In any case, broad or universal mandatory pension plans can only be introduced by political legislators or through agreements between trade unions and employers associations, and such decision makers may apply a different weighting of participant types.

We concluded from Table 2 that a wide range of plan designs improve the welfare of both stock-avoiders and procrastinators with preference parameters $\text{RRA} = (\text{EIS})^{-1} = 4$, and this remains true when we vary these parameter values. The key challenge is to identify a design acceptable to rational individuals with low values of RRA and EIS as they dislike plans with moderate or high contribution rates, low

[24]Identifying ψ from observed decisions is difficult, and the existing empirical studies arrive at very different estimates. While the long-run risk literature has determined an EIS-value for a representative agent that could explain the equity premium puzzle, it is less clear what reasonable EIS-values are for individual agents in a life-cycle model with income and mortality risk. The life-cycle analysis of Gomes and Michaelides (2005) considers EIS equal to 0.2 or 0.5, and the values we apply are very similar. Trabandt and Uhlig (2011, p. 311) state that a value close to 0.5 is the general consensus.

stock weights, and the solidary feature, which goes against plan designs preferred by the irrational individuals. Hence, we consider options allowing the individuals to actively choose to deviate from the default design. According to experiences from similar products and plans, only few individuals will consider deviating, and presumably mostly rational individuals. Based on Table 2, we keep IP4 as the default investment policy, but let the individual actively choose any of the other policies IP1-IP7 from the menu, cf. Dahlquist et al. (2018). With RRA $= 2$, more stocks are desired, and the rational individual would go for IP7.[25] As suggested by Table 2, the investment policy has only a modest welfare effect, though. Our previous analysis showed that individuals with RRA $= (\text{EIS})^{-1} = 4$ highly appreciate the solidary feature of the pension plan, so we take this feature to be the default choice. However, a rational individual with low RRA and EIS prefers less annuitization and dislikes the full annuitization embedded in a solidary pension plan. Therefore, we allow plan participants to actively select a personal plan instead of a solidary plan (in addition to selecting investment policy), and the rational individual with RRA $= 2$ and EIS $= 1/6, 1/4$ would do so. None of the irrational individuals considered would experience welfare losses if they should actively choose another investment policy or select a personal plan.[26]

Table 4 summarizes the results of our analysis. If contributions are initiated at age 25, the largest possible contribution rate generating positive gains to all 27 types is 6%, cf. Panel A. Without any options, this plan would lead to welfare losses for some rational individuals, e.g. a loss of 1.9% with RRA 2 and EIS 1/6. The option to choose a different investment policy reduces the loss to 1.6%. Also adding the option to select a personal instead of a solidary plan results in a gain of 0.2%. The plan leads to sizeable welfare gains for stock avoiders and especially procrastinators, as expected, but also for rational participants with RRA 4 or 6. The average welfare gain across the 27 participants is 15.2%, based on the plan with both options and assuming than none of the irrational participants exercises the options. Appendix B presents additional results showing that a slightly higher contribution rate would increase the average welfare gain—to 15.7% with 7% contributions—but lead to a welfare loss for the rational participant with low RRA and EIS. Lower contribution

[25]With RRA $= 4$, IP4 is the optimal policy. With RRA $= 6$, a switch to IP2 is optimal and improves welfare marginally.

[26]In fact, the stock avoider with RRA $= 2$ and EIS $= 1/6, 1/4$ also has a slightly higher utility for the personal plan variant.

	$\gamma=2$			$\gamma=4$			$\gamma=6$		
	$\psi=\frac{1}{6}$	$\psi=\frac{1}{4}$	$\psi=\frac{1}{2}$	$\psi=\frac{1}{6}$	$\psi=\frac{1}{4}$	$\psi=\frac{1}{2}$	$\psi=\frac{1}{6}$	$\psi=\frac{1}{4}$	$\psi=\frac{1}{2}$
Panel A: 6% from age 25 (Average gain: 15.2)									
Stock avoider	2.1	2.9	5.6	10.5	11.3	12.7	12.2	12.6	12.5
Procrastinator	7.9	9.1	11.0	32.2	42.2	40.2	35.6	58.1	63.1
Rational									
- no options	-1.9	-1.1	1.3	3.1	3.9	5.0	4.3	4.8	5.1
- choose IP	-1.6	-0.8	1.8	3.1	3.9	5.0	4.4	4.9	5.1
- choose pers/soli	0.2	0.4	1.8	3.1	3.9	5.0	4.4	4.9	5.1
Panel B: 9% from age 30 (Average gain: 15.8)									
Stock avoider	2.4	3.1	5.6	11.1	11.7	12.6	12.5	12.7	12.3
Procrastinator	8.4	9.7	9.6	34.1	43.6	40.1	39.0	61.2	64.1
Rational									
- no options	-1.4	-0.8	1.3	3.9	4.5	5.3	5.0	5.2	5.3
- choose IP	-1.1	-0.4	1.9	3.9	4.5	5.3	5.1	5.3	5.4
- choose pers/soli	0.0	0.2	1.9	3.9	4.5	5.3	5.1	5.3	5.4
Panel C: 12% from age 35 (Average gain: 15.4)									
Stock avoider	2.6	3.2	5.6	10.8	11.3	12.1	11.8	11.9	11.5
Procrastinator	8.4	9.7	9.8	33.5	42.9	39.7	37.7	59.6	62.4
Rational									
- no options	-1.0	-0.5	1.5	4.0	4.5	5.2	4.9	5.1	5.2
- choose IP	-0.6	-0.1	2.1	4.0	4.5	5.2	5.0	5.2	5.3
- choose pers/soli	0.1	0.3	2.1	4.0	4.5	5.2	5.0	5.2	5.3
Panel D: 17% from age 40 (Average gain: 14.9)									
Stock avoider	2.4	2.9	5.2	10.3	10.8	11.5	11.2	11.3	10.8
Procrastinator	7.8	9.1	9.6	32.5	41.6	38.8	37.4	58.1	60.3
Rational									
- no options	-0.8	-0.4	1.4	4.0	4.4	5.2	5.0	5.1	5.2
- choose IP	-0.5	0.0	2.1	4.0	4.4	5.2	5.0	5.2	5.2
- choose pers/soli	0.1	0.3	2.1	4.0	4.4	5.2	5.0	5.2	5.2

Table 4: **Welfare effects on heterogeneous participants.** The table shows percentage welfare gains from mandatory enrolment in different pension plans. The plans are all solidary, follows investment policy IP4, and have constant expected payouts through retirement. For the rational individuals, the second-to-last line in each panel reports the welfare effect when the individual is allowed, at the initial date, to replace the default investment policy IP4 with any of the specified policies IP1-IP7. In the last line in each panel, the rational individual is given the additional option to choose a personal plan ($I = 0$) instead of the default solidary feature ($I = 1$). The average gain reported is the simple average of the gains for the nine stock avoiders, the nine procrastinators, and the nine rational individuals having both options. Except for γ and ψ, the baseline parameter values listed in Table 1 are applied.

rates lead to a lower average gain.

Panels B–D show the best plans with contributions from age 30, 35, and 40. In each case, the best plan involves the largest contribution rate that leads to positive gains for all particpants, and lower contributions rates generate lower average gains. The largest average gain of 15.8%—or about $233,000 in present value terms—is obtained with a 9% contribution rate from age 30. Across the 27 participant types, this plan beats the '6% from age 25' plan for 21 participants, the '12% from age 35' plan for 19, and the '17% from age 40' plan for 22 participants. The reported average weighs the 27 participant types evenly. If more than a third of individuals are procrastinating, the corresponding weighted average would provide further support for the '9% from age 30' plan.

Larger average gains may be attained by introducing additional options that are valuable particularly to rational individuals with low RRA and EIS, since this may allow a larger contribution rate which would increase the gains of irrational participants. However, it is important that the options do not clearly tempt irrational individuals to make harmful active decisions away from the default design. For example, while the option to choose a lower contribution rate than the stated default rate of, say, 9% may improve the welfare of rational individuals with low RRA and EIS, this might lead young procrastinators to do the same thing which would reduce their welfare gain substantially. The exploration of additional optional features is left for future research.

6 Reduction in Social Security benefits

The welfare implications of a mandatory retirement savings program depend on the assumed benefits from Social Security. The above analysis is calibrated to the level of benefits currently paid to retirees. However, according to a 2019 Gallup poll, about two thirds of US workers worry a great deal or a fair amount about the Social Security system.[27] Such concerns seem justified. The Social Security and Medicare Boards of Trustees concludes in their April 2020 report that the system will show net cash outflows from 2021, and by 2035 benefits have to be cut by 24% to make the system sustainable.[28] The alternative to cutting benefits would be to increase

[27]Source: https://news.gallup.com/poll/1693/social-security.aspx, accessed on August 7, 2020.

[28]Source: https://www.ssa.gov/OACT/TR/2020/index.html, accessed on August 7, 2020.

the retirement age or the pay-roll tax rate financing the system or cover the deficits through direct government subsidies.

In this section we consider the implications of a 24% reduction in Social Security benefits for the design of a mandatory pension scheme. Hence, we lower the parameter ζ from 0.45 to 0.342, that is social security benefits are reduced from 45% to 34.2% of pre-retirement income. We keep the retirement age and the tax rates unchanged. We have modeled the possible out-of-pocket medical expenses as fractions of Social Security benefits: $h = 3\%$ and $H = 85\%$ in our baseline parametrization. To keep expenses fixed in dollar terms, a 24% reduction in benefits should thus be accompanied by multiplying h and H by $1/(1 - 0.24) \approx 1.316$. However, to avoid costs exceeding income, we cap H at 0.95, effectively assuming that a positive net income is ensured through other welfare programs or support from family and friends. We then repeat the analysis from the previous section with the new parameter values.

Table 5 shows the welfare implications of the optimal designs when contributions start at age 25, 30, 35, or 40. For each starting age, the optimal contribution rate increases when Social Security benefits are cut: from 6% to 7% for age 25, from 9% to 10% for age 30, from 12% to 14% for age 35, and from 17% to 19% for starting age 40. Based on the average welfare gain across the 27 participant types, the overall optimal scheme still involves contributions from age 30, but now with contributions equaling 10% of income. Except maybe for the rational participants with low RRA and EIS, all participants exhibit larger welfare gains in the case with reduced Social Security, in line with intuition. The average welfare gain is now 17.8% compared to 15.8% in the baseline case, an increase amounting to about $30,000 per person in present value terms. With lower Social Security benefits, a well-designed mandatory pension scheme is even more appreciated.

7 Conclusion

Our quantitative analysis of a rich life-cycle model shows that a well-designed mandatory pension scheme substantially improves the welfare of financially unsophisticated individuals who procrastinate on retirement saving or fail to privately invest in stocks, and at the same time the scheme slightly improves the welfare of rational individuals. Calibrated to US data, our model suggests that the best

	$\gamma = 2$			$\gamma = 4$			$\gamma = 6$		
	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$
Panel A: 7% from age 25 (Average gain: 17.2)									
Stock avoider	3.0	3.9	6.8	11.8	12.7	14.0	13.4	13.8	13.6
Procrastinator	10.1	12.1	15.4	35.1	47.8	47.7	37.6	62.6	71.1
Rational									
- no options	-2.0	-1.1	1.5	3.6	4.5	5.7	5.0	5.5	5.7
- choose IP	-1.7	-0.8	2.1	3.6	4.5	5.7	5.1	5.6	5.9
- choose pers/soli	0.2	0.3	2.1	3.6	4.5	5.7	5.1	5.6	5.9
Panel B: 10% from age 30 (Average gain: 17.8)									
Stock avoider	3.6	4.2	6.9	12.4	12.9	13.9	13.5	13.6	13.2
Procrastinator	10.8	13.0	13.8	36.9	49.1	48.0	39.7	64.9	73.8
Rational									
- no options	-1.2	-0.6	1.7	4.5	5.1	5.9	5.6	5.8	5.9
- choose IP	-0.9	-0.2	2.3	4.5	5.1	5.9	5.7	5.9	6.0
- choose pers/soli	0.1	0.2	2.3	4.5	5.1	5.9	5.7	5.9	6.0
Panel C: 14% from age 35 (Average gain: 17.6)									
Stock avoider	3.5	4.1	6.8	12.1	12.6	13.4	13.0	13.1	12.6
Procrastinator	10.6	12.8	13.9	36.6	48.6	47.6	40.1	64.5	72.5
Rational									
- no options	-1.0	-0.4	1.8	4.6	5.1	6.0	5.7	5.9	6.0
- choose IP	-0.6	0.0	2.4	4.6	5.1	6.0	5.8	6.0	6.1
- choose pers/soli	0.0	0.2	2.4	4.6	5.1	6.0	5.8	6.0	6.1
Panel D: 19% from age 40 (Average gain: 16.9)									
Stock avoider	3.4	4.0	6.5	11.6	12.0	12.6	12.2	12.3	11.7
Procrastinator	10.0	12.3	13.9	35.3	46.9	46.6	38.7	61.9	69.8
Rational									
- no options	-0.6	-0.2	1.8	4.6	5.1	5.9	5.6	5.8	5.8
- choose IP	-0.3	0.3	2.5	4.6	5.1	5.9	5.7	5.9	5.9
- choose pers/soli	0.1	0.3	2.5	4.6	5.1	5.9	5.7	5.9	5.9

Table 5: **Welfare effects on heterogeneous participants: Social Security cuts.** The table shows percentage welfare gains from mandatory enrolment in different pension plans when the Social Security benefit multiplier ζ is lowered from 0.45 to 0.342. The plans are all solidary, follows investment policy IP4, and have constant expected payouts through retirement. For the rational individuals, the second-to-last line in each panel reports the welfare effect when the individual is allowed, at the initial date, to replace the default investment policy IP4 with any of the specified policies IP1-IP7. In the last line in each panel, the rational individual is given the additional option to choose a personal plan ($I = 0$) instead of the default solidary feature ($I = 1$). The average gain reported is the simple average of the gains for the nine stock avoiders, the nine procrastinators, and the nine rational individuals having both options. The medical cost parameters h and H are adjusted as explained in the text. For other parameters, the baseline values listed in Table 1 are applied.

scheme features (i) mandatory contributions of 9% of income from age 30 until retirement, (ii) a glidepath investment policy with the stock weight starting at 100%, then sloping down to 40% during the last 15 years before retirement, and staying at 40% through retirement, (iii) tax-exempt returns on pension savings, and (iv) automatic life-long annuitization. Across 27 types of scheme participants, such a scheme improves the welfare of all 27, with an average improvement corresponding to $233,000 per person in present value terms. The largest welfare gains are captured by individuals procrastinating on saving, but also individuals not participating in the stock market make substantial gains from this mandatory scheme. We also show that with the projected cut in Social Security benefits, the contribution rate should be raised to 10%, and that the mandatory savings plan is then even more appreciated with the average gain increasing to $263,000.

While our life-cycle model is rich, a number of relevant and potentially important extensions of our setting easily come to mind. First, we ignore housing which is a key concern of households who can invest in residential real estate, enjoy the benefits from living in the home, and—if home prices increase or mortgage debt is paid down—accumulate wealth in terms of home equity. Note, however, that the welfare implications of a pension plan and its optimal design might then be substantially different for homeowners than for homerenters. Second, while standard in the literature, our model of labor income dynamics ignores unemployment risk, and recent empirical studies have identified various other inadequacies of the model (e.g. Guvenen, Karahan, Ozkan, and Song, 2019). Third, we assume illiquid pension savings, but it would be interesting to allow hardship withdrawals under specified circumstances, maybe at some penalty rate. Other extensions would be to allow for partial annuitization of the pension savings, for non-proportional taxation of returns and income, and for time-varying investment opportunities. Note, however, that any of these extensions induces an additional state variable which further complicates and prolongs the numerical solution.

Overall, our analysis supports the introduction of a broad, carefully designed, mandatory retirement saving scheme, but ignores some potential general equilibrium effects. As pensions savings crowd out some private savings, the zero tax rate on pension returns lowers the total taxes collected from financial returns. In fact, a broad mandatory pension scheme could be welfare improving even with a positive pension return tax rate, and with the increase in total savings it may be possible to obtain an overall return tax revenue as large as in the case without mandatory saving. Also, with the higher retirement income generated by a broad mandatory plan, fewer retirees will require tax-financed welfare payments or other forms of public support. Our welfare calculations assume that the level of the riskfree rate and the expected excess return and volatility of the stock market do not change upon the introduction of the mandatory scheme. Any such changes could also have ramifications for firms' cost of capital and thus overall production and labor market conditions. We leave a complete general equilibrium analysis to future research.

A Solving the utility maximization problem

A.1 The case with a mandatory pension scheme

The problem involves the three state variables F_t, Y_t, and A_t, but is formulated so that the dimensionality can be reduced by one. Different choices of scaling are possible, but they are not equally convenient for the numerical solution approach, which involves a grid for the scaled state variables. We define the scaled state variables

$$y_t = \frac{\bar{Y}_t}{F_t + \bar{A}_t}, \qquad a_t = \frac{\bar{A}_t}{F_t + \bar{A}_t},$$

where $\bar{Y}_t = (1 - \tau_Y)Y_t$ and $\bar{A}_t = (1 - \tau_Y)A_t$. Here, we are using total (after-tax) savings $F_t + \bar{A}_t$ as denominator, which is going to be relatively independent of the assumed pension scheme since larger contribution rates that generate larger values of $\bar{A}$ tend to be partially compensated by lower private savings and thus lower values of F. We are solving the problem on a grid of points (t, y, a). Note that by definition $a \in [0, 1]$ and $y > 0$, and we impose a small lower bound $y^\ell > 0$ and a suitable upper bound $y^u > y^\ell$ on y_t, and solve the problem on a grid in the space $[t_1, t_M] \times [y^\ell, y^u] \times [0, 1]$; see Appendix A.3 for more information on the numerical implementation.

An alternative would be to use disposable wealth $F_t + \bar{Y}_t$ as the denominator and the scaled state variables $y'_t = \bar{Y}_t/(F_t + \bar{Y}_t)$, $a'_t = \bar{A}_t/(F_t + \bar{Y}_t)$. However, in this case the denominator and the relevant range for a' would depend heavily on the pension scheme design, which complicates an appropriate definition of a grid for (y', a'). Yet another alternative would be to use $F_t + \bar{Y}_t + \bar{A}_t$ and the scaled state variables $y''_t = \bar{Y}_t/(F_t + \bar{Y}_t + \bar{A}_t)$, $a''_t = \bar{A}_t/(F_t + \bar{Y}_t + \bar{A}_t)$. However, in this case $y'' + a'' \le 1$, which hinders the use of a square grid for (y'', a'').

Final year. Death is certain at the end of the period ($p_{t_M} = 0$). Since $m_{t_M} = 1$, the bequest is

$$B_{t_M+1} = F_{t_M+1} = (1 - c_{t_M})\tilde{F}_{t_M} R_{F,t_M} = (1 - c_{t_M})(F_{t_M} + \bar{Y}_{t_M} + \bar{A}_{t_M})R_{F,t_M}.$$

The certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_{t_M} &= \left(\mathrm{E}_{t_M}\left[\xi^{\frac{1-\gamma}{\psi-1}} B_{t_M+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} = \xi^{\frac{1}{\psi-1}}\left(1 - c_{t_M}\right)\left(F_{t_M} + \bar{Y}_{t_M} + \bar{A}_{t_M}\right)\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \\
&= \xi^{\frac{1}{\psi-1}}(1 - c_{t_M})(F_{t_M} + \bar{A}_{t_M})(1 + y_{t_M})\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}},
\end{aligned}$$

and the indirect utility is

$$
\begin{aligned}
J_{t_M} &= \max_{c_{t_M},\pi_{t_M}} \left\{ \left(c_{t_M}\tilde{F}_{t_M}\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_{t_M}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&= (F_{t_M} + \bar{A}_{t_M})(1+y_{t_M}) \max_{c_{t_M},\pi_{t_M}} \left\{ c_{t_M}^{1-\frac{1}{\psi}} + \beta\xi^{\frac{1}{\psi}} \left(1-c_{t_M}\right)^{1-\frac{1}{\psi}} \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.
\end{aligned}
$$

Here, the optimal stock weight $\pi^*_{t_M}$ is determined by maximizing $\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{1/(1-\gamma)}$, and then the optimal consumption rate and the indirect utility are given by

$$
c^*_{t_M} = \left(1 + \xi\beta^{\psi}\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{\psi-1}{1-\gamma}}\right)^{-1}, \tag{A.1}
$$

$$
J_{t_M} = (F_{t_M} + \bar{A}_{t_M}) G_{t_M}(y_{t_M}, a_{t_M}), \tag{A.2}
$$

where $\pi^*_{t_M}$ is applied for generating the return and

$$
G_{t_M}(y_{t_M}, a_{t_M}) = (1+y_{t_M}) \left\{ \left(c^*_{t_M}\right)^{1-\frac{1}{\psi}} + \beta\xi^{\frac{1}{\psi}}\left(1-c^*_{t_M}\right)^{1-\frac{1}{\psi}} \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{A.3}
$$

which, in fact, does not depend on a_{t_M}. Note that $c^*_{t_M} \approx 1/(1+\xi)$ so for large values of ξ, only a small fraction of wealth is consumed in the final year while the end-of-year bequest is large. As ξ approaches zero, more of the wealth is consumed and less left for bequest. These observations confirm that ξ measures the strength of the bequest motive.

Non-final years, $t = t_1, \ldots, t_M - 1$. The bequest next year is $B_{t+1} = F_{t+1} + (1-I)\bar{A}_{t+1}$. For an induction argument, we assume that $J_{t+1} = (F_{t+1} + \bar{A}_{t+1})G_{t+1}(y_{t+1}, a_{t+1})$ which implies that the certainty equivalent is

$$
\begin{aligned}
\mathrm{CE}_t &= \Big(p_t \mathrm{E}_t\left[(F_{t+1} + \bar{A}_{t+1})^{1-\gamma} G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma}\right] \\
&\qquad + (1-p_t)\mathrm{E}_t\left[\xi^{\frac{1-\gamma}{\psi-1}}(F_{t+1} + (1-I)\bar{A}_{t+1})^{1-\gamma}\right]\Big)^{\frac{1}{1-\gamma}} \\
&= (F_t + \bar{A}_t)\Bigg(p_t \mathrm{E}_t\left[\left(\tfrac{F_{t+1}+\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma} G_{t+1}(y_{t+1}, a_{t+1})^{1-\gamma}\right] \\
&\qquad + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\tfrac{F_{t+1}+(1-I)\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}\right]\Bigg)^{\frac{1}{1-\gamma}} \equiv (F_t + \bar{A}_t)\mathcal{C}_t(y_t, a_t).
\end{aligned}
$$

Here, the expectation is over the shock ε_{St} to stock prices and (before retirement) the shock ε_{Yt} to income, and we use that (for $k=1$ or $k=1-I$)

$$\frac{F_{t+1}+k\bar{A}_{t+1}}{F_t+\bar{A}_t} = (1-c_t)\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)R_{Ft} + k\left((1-m_t)a_t+\alpha_t y_t\right)R_{At}\,\frac{1}{1-I+Ip_t},$$

$$y_{t+1} = \frac{y_t R_{Yt}}{(1-c_t)\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)R_{Ft} + \left((1-m_t)a_t+\alpha_t y_t\right)R_{At}\frac{1}{1-I+Ip_t}},$$

$$a_{t+1} = \frac{\left((1-m_t)a_t+\alpha_t y_t\right)R_{At}\frac{1}{1-I+Ip_t}}{(1-c_t)\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)R_{Ft} + \left((1-m_t)a_t+\alpha_t y_t\right)R_{At}\frac{1}{1-I+Ip_t}}.$$

The utility recursion (7) implies that

$$\begin{aligned}
J_t &= \max_{c_t,\pi_t}\left\{c_t^{1-\frac{1}{\psi}}\left(F_t+(1-\alpha_t)\bar{Y}_t+m_t\bar{A}_t\right)^{1-\frac{1}{\psi}} + \beta\left(F_t+\bar{A}_t\right)^{1-\frac{1}{\psi}}\mathcal{C}_t(y_t,a_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&= \left(F_t+\bar{A}_t\right)\max_{c_t,\pi_t}\left\{c_t^{1-\frac{1}{\psi}}\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)^{1-\frac{1}{\psi}} + \beta\mathcal{C}_t(y_t,a_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
&\equiv \left(F_t+\bar{A}_t\right)G_t\left(y_t,a_t\right).
\end{aligned}$$

Since the expectation in $\mathcal{C}_t$ involves G_{t+1}, we get the recursion

$$\begin{aligned}
G_t\left(y_t,a_t\right) = \max_{c_t,\pi_t}\Bigg\{ & c_t^{1-\frac{1}{\psi}}\left(1+(1-\alpha_t)y_t-(1-m_t)a_t\right)^{1-\frac{1}{\psi}} \\
& + \beta\Bigg(p_t\mathrm{E}_t\left[\left(\tfrac{F_{t+1}+\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}G_{t+1}(y_{t+1},a_{t+1})^{1-\gamma}\right] \\
& \qquad + (1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\tfrac{F_{t+1}+(1-I)\bar{A}_{t+1}}{F_t+\bar{A}_t}\right)^{1-\gamma}\right]\Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned}$$

We solve this backwards starting with $t=t_M-1$ in which case G_{t_M} is known from (A.3).

A.2 The case without a mandatory pension scheme

Suppose the individual spends a fraction $\eta\in[0,1]$ of his disposable wealth $F_{t_R-1}+\bar{Y}_{t_R-1}$ on purchasing a life-long annuity which provides a payment at the beginning of each year $t_R, t_R+1,\ldots,t_M$ conditional on being alive. The annuity stipulates an assumed interest rate $\tilde{r}$ and a benchmark portfolio. As annuity payments are made, the value of the portfolio is reduced correspondingly. Let V_t denote the value of the portfolio at the beginning of year t, just before the annuity payment is made. We let m_t denote the fraction of the portfolio value paid out, i.e. the annuity pays m_tV_t at the beginning of year t, where

$$m_t = \left(\sum_{s=t}^{t_M}P_{t,s}e^{-\tilde{r}(s-t)}\right)^{-1}, \quad t=t_R,\ldots,t_M.$$

We assume that the amount $\eta\left(F_{t_R-1}+\bar{Y}_{t_R-1}\right)/(1+k)$ is invested in the portfolio immediately after the annuitization decision at the beginning of year t_R-1, where $k \geq 0$ represents costs and profits of the issuer making the annuity less-than-fair for the individual. The payment from the annuity is not taxed as it is financed by the savings of after-tax labor income. The portfolio value at the beginning of year t_R is then

$$V_{t_R}=\frac{\eta}{1+k}\left(F_{t_R-1}+\bar{Y}_{t_R-1}\right)R_{V,t_R-1}p_{t_R-1}^{-1},$$

and the dynamics of the portfolio value are

$$V_{t+1}=V_t(1-m_t)R_{Vt}p_t^{-1}, \quad t=t_R+1,\ldots,t_M,$$

where R_{Vt} is the gross after-tax return on the portfolio, and the term p_t^{-1} represents a transfer of value from deceased customers in year t. We assume the same tax rate τ_F applies to the returns on the annuity portfolio as the returns on non-annuitized private investments so

$$R_{Vt}=\tau_F+(1-\tau_F)\exp\left\{r+w_t\mu_S-\frac{1}{2}w_t^2\sigma_S^2+w_t\sigma_S\varepsilon_{St}\right\},$$

where w_t is the annuity portfolio weight of the stock in year t. As explained in Section 2, the payout is increasing [decreasing] if the realized log after-tax return $\ln R_{Vt}$ is greater [smaller] than the assumed interest rate $\tilde{r}$. A fixed annuity is a special case where $\tilde{r}=\ln\left(\tau_F+(1-\tau_F)(e^r-1)\right)$.

The dynamics of private, non-annuitized wealth are

$$F_{t+1}=\begin{cases}(1-c_t)\left(F_t+\bar{Y}_t+m_tV_t\right)R_{Ft} & \text{for } t=t_R,\ldots,t_M,\\ (1-c_t)(1-\eta)\left(F_t+\bar{Y}_t\right)R_{Ft} & \text{for } t=t_R-1,\\ (1-c_t)\left(F_t+\bar{Y}_t\right)R_{Ft} & \text{for } t=t_1,\ldots,t_R-2.\end{cases}$$

Here $\bar{Y}_t=(1-\tau_Y)Y_t$ and the dynamics of Y are given by (1).

The state variables for this problem are $F_t,\bar{Y}_t$ before retirement and $F_t,\bar{Y}_t,V_t$ in retirement. The problem set up allows us to reduce the dimensionality by one through a scaling. We show below that the indirect utility has the form

$$J_t=\begin{cases}(F_t+V_t)G_t(y_t,v_t) & \text{for } t=t_R,\ldots,t_M,\\ F_tG_t(y_t), & \text{for } t=t_1,\ldots,t_R-1,\end{cases}$$

where

$$y_t = \frac{\bar{Y}_t}{F_t + V_t}, \quad v_t = \frac{V_t}{F_t + V_t},$$

with $V_t = v_t = 0$ for $t = t_1, \dots, t_R - 1$, and G_t and the optimal decisions are determined by backward recursion.

Final year, $t = t_M$. The individual is sure to die at the end of the period, leaving a bequest of

$$B_{t_M+1} = F_{t_M+1} = (1 - c_{t_M}) \left(F_{t_M} + \bar{Y}_{t_M} + V_{t_M}\right) R_{F,t_M},$$

where we have applied $m_{t_M} = 1$. The certainty equivalent is therefore

$$\mathrm{CE}_{t_M} = \left(\mathrm{E}_{t_M}\left[\xi^{\frac{1-\gamma}{\psi-1}} B_{t_M+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} = \xi^{\frac{1}{\psi-1}}(1-c_{t_M})\left(F_{t_M} + V_{t_M}\right)(1+y_{t_M})\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}.$$

The indirect utility is

$$J_{t_M} = \max_{c_{t_M},\pi_{t_M}} \left\{ \left(c_{t_M}[F_{t_M} + \bar{Y}_{t_M} + V_{t_M}]\right)^{1-\frac{1}{\psi}} + \beta\, \mathrm{CE}_{t_M}^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} = \left(F_{t_M} + V_{t_M}\right) G_{t_M}(y_{t_M}, v_{t_M}),$$

where

$$G_{t_M}(y_{t_M}, v_{t_M}) = (1 + y_{t_M}) \max_{c_{t_M},\pi_{t_M}} \left\{ c_{t_M}^{1-\frac{1}{\psi}} + \beta \xi^{\frac{1}{\psi}} (1 - c_{t_M})^{1-\frac{1}{\psi}} \left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

which in fact is independent of v_{t_M}. The optimal portfolio weight $\pi_{t_M}^*$ is determined numerically by maximizing $\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$. The optimal consumption rate is

$$c_{t_M}^* = \left(1 + \xi\beta^{\psi}\left(\mathrm{E}_{t_M}\left[R_{F,t_M}^{1-\gamma}\right]\right)^{\frac{\psi-1}{1-\gamma}}\right)^{-1}.$$

In retirement, $t = t_R, t_R + 1, \dots, t_M - 1$. The bequest if dying at the beginning of year $t+1$ is $B_{t+1} = F_{t+1}$. With $J_{t+1} = (F_{t+1} + V_{t+1})\, G_{t+1}(y_{t+1}, v_{t+1})$, the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_t &= \left(p_t \mathrm{E}_t\left[(F_{t+1} + V_{t+1})^{1-\gamma} G_{t+1}(y_{t+1}, v_{t+1})^{1-\gamma}\right] + (1 - p_t)\mathrm{E}_t\left[\xi^{\frac{1-\gamma}{\psi-1}} F_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \\
&= (F_t + V_t)\left(p_t \mathrm{E}_t\left[\left(\tfrac{F_{t+1}+V_{t+1}}{F_t+V_t}\right)^{1-\gamma} G_{t+1}(y_{t+1}, v_{t+1})^{1-\gamma}\right] + (1 - p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\tfrac{F_{t+1}}{F_t+V_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \\
&\equiv (F_t + V_t)\mathcal{C}_t(y_t, v_t).
\end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices, and we use that

$$\begin{aligned}
\frac{F_{t+1}+V_{t+1}}{F_t+V_t} &= (1-c_t)\left(1+y_t-(1-m_t)v_t\right)R_{Ft}+(1-m_t)v_tR_{Vt}p_t^{-1},\\
\frac{F_{t+1}}{F_t+V_t} &= (1-c_t)\left(1+y_t-(1-m_t)v_t\right)R_{Ft},\\
y_{t+1} &= \frac{y_tR_{Yt}}{(1-c_t)\left(1+y_t-(1-m_t)v_t\right)R_{Ft}+(1-m_t)v_tR_{Vt}p_t^{-1}},\\
v_{t+1} &= \frac{(1-m_t)v_tR_{Vt}p_t^{-1}}{(1-c_t)\left(1+y_t-(1-m_t)v_t\right)R_{Ft}+(1-m_t)v_tR_{Vt}p_t^{-1}}
\end{aligned}$$

with $R_{Yt}=1$. The indirect utility is

$$\begin{aligned}
J_t &= \max_{c_t,\pi_t}\left\{c_t^{1-\frac{1}{\psi}}(F_t+\bar{Y}_t+m_tV_t)^{1-\frac{1}{\psi}}+\beta\left(F_t+V_t\right)^{1-\frac{1}{\psi}}\mathcal{C}_t(y_t,v_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\\
&= (F_t+V_t)\max_{c_t,\pi_t}\left\{c_t^{1-\frac{1}{\psi}}(1+y_t-(1-m_t)v_t)^{1-\frac{1}{\psi}}+\beta\mathcal{C}_t(y_t,v_t)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\equiv(F_t+V_t)G_t(y_t,v_t).
\end{aligned}$$

Note that G satisfies the recursion

$$\begin{aligned}
G_t\left(y_t,v_t\right) = \max_{c_t,\pi_t}\Bigg\{ & c_t^{1-\frac{1}{\psi}}(1+y_t-(1-m_t)v_t)^{1-\frac{1}{\psi}}\\
&+\beta\left(p_t\mathrm{E}_t\left[\left(\tfrac{F_{t+1}+V_{t+1}}{F_t+V_t}\right)^{1-\gamma}G_{t+1}(y_{t+1},v_{t+1})^{1-\gamma}\right]+(1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\tfrac{F_{t+1}}{F_t+V_t}\right)^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned}$$

The optimal consumption rate c_t^* and the optimal portfolio weight π_t^* are determined numerically.

At retirement, $t=t_R-1$. The bequest if dying at the beginning of year t_R is $B_{t_R}=F_{t_R}$. With $J_{t_R}=\left(F_{t_R}+V_{t_R}\right)G_{t_R}(y_{t_R},v_{t_R})$, the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_{t_R-1} &= \left(p_{t_R-1}\mathrm{E}_{t_R-1}\left[(F_{t_R}+V_{t_R})^{1-\gamma}G_{t_R}(y_{t_R},v_{t_R})^{1-\gamma}\right]+(1-p_{t_R-1})\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_{t_R-1}\left[F_{t_R}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\\
&= F_{t_R-1}\Bigg(p_{t_R-1}\mathrm{E}_{t_R-1}\left[\left(\tfrac{F_{t_R}+V_{t_R}}{F_{t_R-1}}\right)^{1-\gamma}G_{t_R}(y_{t_R},v_{t_R})^{1-\gamma}\right]\\
&\qquad+(1-p_{t_R-1})\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_{t_R-1}\left[\left(\tfrac{F_{t_R}}{F_{t_R-1}}\right)^{1-\gamma}\right]\Bigg)^{\frac{1}{1-\gamma}}\\
&\equiv F_{t_R-1}\mathcal{C}_{t_R-1}(y_{t_R-1}).
\end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices, and we use that

$$\begin{aligned}
\frac{F_{t_R}+V_{t_R}}{F_{t_R-1}} &= (1+y_{t_R-1})\left[(1-c_{t_R-1})(1-\eta)R_{F,t_R-1}+\frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}\right],\\
\frac{F_{t_R}}{F_{t_R-1}} &= (1-c_{t_R-1})(1-\eta)(1+y_{t_R-1})R_{F,t_R-1},\\
y_{t_R} &= \frac{\zeta y_{t_R-1}}{(1+y_{t_R-1})\left[(1-c_{t_R-1})(1-\eta)R_{F,t_R-1}+\frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}\right]},\\
v_{t_R} &= \frac{\frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}}{(1-c_{t_R-1})(1-\eta)R_{F,t_R-1}+\frac{\eta}{1+k}R_{V,t_R-1}p_{t_R-1}^{-1}}.
\end{aligned}$$

The indirect utility is

$$\begin{aligned}
J_{t_R-1} &= \max_{c_{t_R-1},\pi_{t_R-1},\eta}\left\{c_{t_R-1}^{1-\frac{1}{\psi}}(F_{t_R-1}+\bar{Y}_{t_R-1})^{1-\frac{1}{\psi}}+\beta F_{t_R-1}^{1-\frac{1}{\psi}}\mathcal{C}_{t_R-1}(y_{t_R-1})^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\\
&= F_{t_R-1}\max_{c_{t_R-1},\pi_{t_R-1},\eta}\left\{c_{t_R-1}^{1-\frac{1}{\psi}}(1+y_{t_R-1})^{1-\frac{1}{\psi}}+\beta\mathcal{C}_{t_R-1}(y_{t_R-1})^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}\\
&\equiv F_{t_R-1}G_{t_R-1}(y_{t_R-1}).
\end{aligned}$$

Note that G satisfies the recursion

$$\begin{aligned}
G_{t_R-1}(y_{t_R-1}) = \max_{\substack{\eta,c_{t_R-1}\\ \pi_{t_R-1}}}\Bigg\{c_{t_R-1}^{1-\frac{1}{\psi}}(1+y_{t_R-1})^{1-\frac{1}{\psi}}+\beta\Bigg(p_{t_R-1}\mathrm{E}_{t_R-1}\left[\left(\frac{F_{t_R}+V_{t_R}}{F_{t_R-1}}\right)^{1-\gamma}G_{t_R}(y_{t_R},v_{t_R})^{1-\gamma}\right]\\
+(1-p_{t_R-1})\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_{t_R-1}\left[\left(\frac{F_{t_R}}{F_{t_R-1}}\right)^{1-\gamma}\right]\Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\Bigg\}^{\frac{1}{1-\frac{1}{\psi}}}.
\end{aligned}$$

The optimal annuitization rate η^*, the optimal consumption rate $c_{t_R-1}^*$, and the optimal portfolio weight $\pi_{t_R-1}^*$ are determined by numerical maximization.

Before retirement, $t=t_1,t_1+1,\ldots,t_R-2$. The bequest if dying at the beginning of year $t+1$ is $B_{t+1}=F_{t+1}$. With $J_{t+1}=F_{t+1}G_{t+1}(y_{t+1})$, the certainty equivalent is

$$\begin{aligned}
\mathrm{CE}_t &= \left(p_t\mathrm{E}_t\left[F_{t+1}^{1-\gamma}G_{t+1}(y_{t+1})^{1-\gamma}\right]+(1-p_t)\mathrm{E}_t\left[\xi^{\frac{1-\gamma}{\psi-1}}F_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\\
&= F_t\left(p_t\mathrm{E}_t\left[\left(\frac{F_{t+1}}{F_t}\right)^{1-\gamma}G_{t+1}(y_{t+1})^{1-\gamma}\right]+(1-p_t)\xi^{\frac{1-\gamma}{\psi-1}}\mathrm{E}_t\left[\left(\frac{F_{t+1}}{F_t}\right)^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\equiv F_t\mathcal{C}_t(y_t).
\end{aligned}$$

Here, the expectation is over the shock ε_{St} to stock prices and the shock ε_{Yt} to income, and we use that

$$\frac{F_{t+1}}{F_t}=(1-c_t)(1+y_t)R_{Ft},\quad y_{t+1}=\frac{y_tR_{Yt}}{(1-c_t)(1+y_t)R_{Ft}}.$$

The indirect utility is

$$J_t = \max_{c_t,\pi_t} \left\{ c_t^{1-\frac{1}{\psi}} (F_t + \bar{Y}_t)^{1-\frac{1}{\psi}} + \beta F_t^{1-\frac{1}{\psi}} \mathcal{C}_t(y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}$$
$$= F_t \max_{c_t,\pi_t} \left\{ c_t^{1-\frac{1}{\psi}} (1 + y_t)^{1-\frac{1}{\psi}} + \beta \mathcal{C}_t(y_t)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \equiv F_t G_t(y_t).$$

Note that G satisfies the recursion

$$G_t\,(y_t) = \max_{c_t,\pi_t} \left\{ c_t^{1-\frac{1}{\psi}} (1 + y_t)^{1-\frac{1}{\psi}} + \beta \Bigg(p_t \mathrm{E}_t \left[\left(\tfrac{F_{t+1}}{F_t} \right)^{1-\gamma} G_{t+1}(y_{t+1})^{1-\gamma} \right] + (1-p_t) \xi^{\frac{1-\gamma}{\psi-1}} \mathrm{E}_t \left[\left(\tfrac{F_{t+1}}{F_t} \right)^{1-\gamma} \right] \Bigg)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

The optimal consumption rate c_t^* and the optimal portfolio weight π_t^* are determined numerically.

In the situation where the individual does not have access to annuitization at retirement, then $\eta = 0$ in the above derivations and the state variable $v_t \equiv 0$ drops out.

A.3 The numerical solution approach

Both in the case with and the case without a mandatory pensions scheme, the scaled state variable y is varying considerably over the working phase of life. With typical values of parameters and initial conditions, y starts out very high as annual income tends to be large relative to financial wealth for young individuals. As wealth accumulates over life, the value of y typically drops considerably approaching retirement. Such variations can be problematic when implementing the dynamic programming approach to utility maximization on an equidistant and relatively sparse grid.[29] Hence, we define

$$\hat{y}_t = y_t \exp\{-k_y(t_R - 1 - t)^+\}$$

and form the grid using $\hat{y}$ instead of y. The scaled variable $\hat{y}$ is more stable over life than y with an appropriate choice of k_y, which depends on the values of parameters, initial conditions, the contribution rate and starting age, as well as the assume degree of financial sophistication. Near the baseline values, $k_y = 0.012 Y_{t_1}/F_{t_1}$ work well for a rational individual and a stock avoider, whereas 0.004 or so works well for a procrastinator.

We use a 21×21 equidistant grid on $(\hat{y}, a) \in [0.01, 0.01 + (\hat{y}_{t_1} - 0.01)/0.8] \times [0, 1]$. We first solve backwards in time for optimal decisions and the indirect utility on the grid,

[29] In retirement, both wealth and net income decrease, leaving less systematic variation in y.

and then we simulate forward in time and make sure that the scaled state variables along almost all paths stay nicely within the grid boundaries.

The backward recursions for the function G_t involve an expectation of a function of one or two standard normally distributed random variables. We approximate this expectation by the use of Gauss-Hermite quadrature. The continuous distribution of each standard normal random variable is approximated by a discrete distribution involving some number n of possible values and associated weights. In the one-dimensional case, we would like to calculate $\mathrm{E}[h(\varepsilon_1)]$ where $\varepsilon_1 \sim N(0,1)$. Then the approximation is

$$\mathrm{E}[h(\varepsilon_1)] \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i h(\sqrt{2} x_i),$$

where x_i is node i, w_i the associated weight, and π is the constant 3.14159.... The ratio $p_i = w_i/\sqrt{\pi}$ is effectively the probability assigned to $z_i = \sqrt{2}x_i$. In the two-dimensional case, we would like to calculate $\mathrm{E}[h(\varepsilon_1, \varepsilon_2)]$ where $\varepsilon_1, \varepsilon_2 \sim N(0,1)$ are independent. Then the approximation is

$$\mathrm{E}[h(\varepsilon_1, \varepsilon_2)] \approx \frac{1}{\pi} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j h(\sqrt{2} x_i, \sqrt{2} x_j),$$

The nodes and weights can be calculated with the socalled Golub-Welsch method. We use $n = 9$ nodes as, e.g., Blake et al. (2014, App. A), and in this case the numbers are:

i	x_i	w_i	y_i	p_i
1	-3.19099	3.96070×10^{-5}	-4.51274	2.23459×10^{-5}
2	-2.26658	0.00494362	-3.20543	0.00278914
3	-1.46855	0.0884745	-2.07684	0.0499164
4	-0.723551	0.432652	-1.02326	0.244098
5	0	0.720235	0	0.406349
6	0.723551	0.432652	1.02326	0.244098
7	1.46855	0.088475	2.07684	0.0499164
8	2.26658	0.00494362	3.20543	0.00278914
9	3.19099	3.96070×10^{-5}	4.51274	2.23459×10^{-5}

The values of the state variables corresponding to $\sqrt{2}x_i$ and $\sqrt{2}x_j$ do typically not match points in the grid for the state variable. If outside the grid boundaries, we use the value at the nearest boundary. If inside the grid boundaries, we use linear interpolation.

B Additional results

This section provides additional results on the welfare implications of selected pension plan designs on a set of 27 individuals with heterogeneous preferences and financial sophistication.

Table B.1 considers three designs with contributions starting immediately at age 25 with a contribution rate of either 5%, 6%, or 7%. In all cases, the default investment policy is IP4, and the plans are solidary with lifelong payouts that are expected to be constant. With a 7% contribution rate (or higher), the plan leads to a welfare loss for a rational individual with RRA = 2 and EIS = 1/6, even when the two optionality features are added, so this design is not acceptable according to our criteria. The plans with 5% or 6% contribution rates improve welfare for all 27 types of individuals, but the plan with 6% contribution has a higher average welfare gain of 15.2%, compared to 14.3% for the plan with 5% contributions. Contribution rates lower than 5% lead to an ever lower average gain. Hence, the 6%-plan is the best plan with contributions starting from age 25. Tables B.2–B.4 contain similar results with contributions starting at age 30, 35, and 40, respectively. The best plan for each starting age is included in Table 4.

	$\gamma = 2$			$\gamma = 4$			$\gamma = 6$		
	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$
Panel A: 5% (Average gain: 14.3)									
Stock avoider	2.6	3.2	5.6	10.1	10.9	11.9	11.3	11.7	11.5
Procrastinator	7.8	9.2	11.0	30.0	40.2	38.9	30.1	52.4	60.4
Rational									
- no options	-1.3	-0.6	1.5	3.0	3.7	4.5	3.8	4.2	4.4
- choose IP	-1.0	-0.2	2.0	3.0	3.7	4.5	3.9	4.2	4.4
- choose pers/soli	0.5	0.6	2.0	3.0	3.7	4.5	3.9	4.2	4.4
Panel B: 6% (Average gain: 15.2)									
Stock avoider	2.1	2.9	5.6	10.5	11.3	12.7	12.2	12.6	12.5
Procrastinator	7.9	9.1	11.0	32.2	42.2	40.2	35.6	58.1	63.1
Rational									
- no options	-1.9	-1.1	1.3	3.1	3.9	5.0	4.3	4.8	5.1
- choose IP	-1.6	-0.8	1.8	3.1	3.9	5.0	4.4	4.9	5.1
- choose pers/soli	0.2	0.4	1.8	3.1	3.9	5.0	4.4	4.9	5.1
Panel C: 7% (Average gain: 15.7)									
Stock avoider	1.5	2.3	5.4	10.5	11.5	13.1	12.7	13.2	13.1
Procrastinator	7.6	9.0	11.4	33.4	43.2	40.9	39.9	62.1	64.9
Rational									
- no options	-2.7	-1.7	1.0	2.9	3.8	5.1	4.6	5.2	5.5
- choose IP	-2.4	-1.4	1.5	2.9	3.8	5.1	4.8	5.4	5.6
- choose pers/soli	-0.1	0.1	1.5	2.9	3.8	5.1	4.8	5.4	5.6

Table B.1: **Welfare effects on heterogeneous participants: Contributions from age 25.** The table shows percentage welfare gains from mandatory enrolment in different pension plans. The plans are all solidary, follows investment policy IP4, and have constant expected payouts through retirement. For the rational individuals, the second-to-last line in each panel reports the welfare effect when the individual is allowed, at the initial date, to replace the default investment policy IP4 with any of the specified policies IP1-IP7. In the last line in each panel, the rational individual is given the additional option to choose a personal plan ($I = 0$) instead of the default solidary feature ($I = 1$). The average gain reported is the simple average of the gains for the nine stock avoiders, the nine procrastinators, and the nine rational individuals having both options. Except for γ and ψ, the baseline parameter values listed in Table 1 are applied.

	$\gamma = 2$			$\gamma = 4$			$\gamma = 6$		
	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$
Panel A: 8% (Average gain: 15.3)									
Stock avoider	2.8	3.4	5.7	10.9	11.5	12.3	11.9	12.1	11.7
Procrastinator	8.6	9.9	9.9	33.0	42.7	39.6	35.3	58.0	62.7
Rational									
- no options	-1.0	-0.5	1.5	3.8	4.3	5.0	4.6	4.8	4.9
- choose IP	-0.7	-0.1	2.1	3.8	4.3	5.0	4.7	4.8	4.9
- choose pers/soli	0.3	0.4	2.1	3.8	4.3	5.0	4.7	4.8	4.9
Panel B: 9% (Average gain: 15.8)									
Stock avoider	2.4	3.1	5.6	11.1	11.7	12.6	12.5	12.7	12.3
Procrastinator	8.4	9.7	9.6	34.1	43.6	40.1	39.0	61.2	64.1
Rational									
- no options	-1.4	-0.8	1.3	3.9	4.5	5.3	5.0	5.2	5.3
- choose IP	-1.1	-0.4	1.9	3.9	4.5	5.3	5.1	5.3	5.4
- choose pers/soli	0.0	0.2	1.9	3.9	4.5	5.3	5.1	5.3	5.4
Panel C: 10% (Average gain: 16.1)									
Stock avoider	2.0	2.6	5.4	11.0	11.7	12.8	12.9	13.1	12.7
Procrastinator	8.2	9.4	9.2	37.7	44.0	40.4	41.8	63.6	65.1
Rational									
- no options	-1.9	-1.3	1.1	3.9	4.5	5.5	5.4	5.6	5.7
- choose IP	-1.6	-0.9	1.7	3.9	4.5	5.5	5.5	5.8	5.8
- choose pers/soli	-0.3	-0.0	1.7	3.9	4.5	5.5	5.5	5.8	5.8

Table B.2: **Welfare effects on heterogeneous participants: Contributions from age 30.** The table shows percentage welfare gains from mandatory enrolment in different pension plans. The plans are all solidary, follows investment policy IP4, and have constant expected payouts through retirement. For the rational individuals, the second-to-last line in each panel reports the welfare effect when the individual is allowed, at the initial date, to replace the default investment policy IP4 with any of the specified policies IP1-IP7. In the last line in each panel, the rational individual is given the additional option to choose a personal plan ($I = 0$) instead of the default solidary feature ($I = 1$). The average gain reported is the simple average of the gains for the nine stock avoiders, the nine procrastinators, and the nine rational individuals having both options. Except for γ and ψ, the baseline parameter values listed in Table 1 are applied.

	$\gamma = 2$			$\gamma = 4$			$\gamma = 6$		
	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$
Panel A: 11% (Average gain: 15.0)									
Stock avoider	2.9	3.4	5.7	10.7	11.1	11.8	11.3	11.4	11.0
Procrastinator	8.5	9.8	10.0	32.7	42.3	39.3	34.9	57.3	61.4
Rational									
- no options	-0.7	-0.2	1.6	3.9	4.3	4.9	4.6	4.7	4.8
- choose IP	-0.3	0.2	2.2	3.9	4.3	4.9	4.6	4.8	4.9
- choose pers/soli	0.3	0.5	2.2	3.9	4.3	4.9	4.6	4.8	4.9
Panel B: 12% (Average gain: 15.4)									
Stock avoider	2.6	3.2	5.6	10.8	11.3	12.1	11.8	11.9	11.5
Procrastinator	8.4	9.7	9.8	33.5	42.9	39.7	37.7	59.6	62.4
Rational									
- no options	-1.0	-0.5	1.5	4.0	4.5	5.2	4.9	5.1	5.2
- choose IP	-0.6	-0.1	2.1	4.0	4.5	5.2	5.0	5.2	5.3
- choose pers/soli	0.1	0.3	2.1	4.0	4.5	5.2	5.0	5.2	5.3
Panel C: 13% (Average gain: 15.6)									
Stock avoider	2.3	2.9	5.4	10.8	11.3	12.2	12.1	12.3	11.9
Procrastinator	8.1	9.4	9.5	33.9	43.1	39.8	40.0	61.4	63.1
Rational									
- no options	-1.3	-0.8	1.3	4.0	4.5	5.4	5.2	5.4	5.5
- choose IP	-1.0	-0.4	1.9	4.0	4.5	5.4	5.3	5.5	5.6
- choose pers/soli	-0.1	0.1	1.9	4.0	4.5	5.4	5.3	5.5	5.6

Table B.3: **Welfare effects on heterogeneous participants: Contributions from age 35.** The table shows percentage welfare gains from mandatory enrolment in different pension plans. The plans are all solidary, follows investment policy IP4, and have constant expected payouts through retirement. For the rational individuals, the second-to-last line in each panel reports the welfare effect when the individual is allowed, at the initial date, to replace the default investment policy IP4 with any of the specified policies IP1-IP7. In the last line in each panel, the rational individual is given the additional option to choose a personal plan ($I = 0$) instead of the default solidary feature ($I = 1$). The average gain reported is the simple average of the gains for the nine stock avoiders, the nine procrastinators, and the nine rational individuals having both options. Except for γ and ψ, the baseline parameter values listed in Table 1 are applied.

	$\gamma = 2$			$\gamma = 4$			$\gamma = 6$		
	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$	$\psi = \frac{1}{6}$	$\psi = \frac{1}{4}$	$\psi = \frac{1}{2}$
Panel A: 16% (Average gain: 14.7)									
Stock avoider	2.6	3.1	5.4	10.3	10.7	11.3	10.8	10.9	10.5
Procrastinator	7.9	9.3	9.8	32.1	41.4	38.7	35.4	56.7	59.8
Rational									
- no options	-0.6	-0.2	1.5	3.9	4.3	5.0	4.7	4.9	5.0
- choose IP	-0.3	0.2	2.2	3.9	4.3	5.0	4.8	4.9	5.0
- choose pers/soli	0.3	0.4	2.2	3.9	4.3	5.0	4.8	4.9	5.0
Panel B: 17% (Average gain: 14.9)									
Stock avoider	2.4	2.9	5.2	10.3	10.8	11.5	11.2	11.3	10.8
Procrastinator	7.8	9.1	9.6	32.5	41.6	38.8	37.4	58.1	60.3
Rational									
- no options	-0.8	-0.4	1.4	4.0	4.4	5.2	5.0	5.1	5.2
- choose IP	-0.5	0.0	2.1	4.0	4.4	5.2	5.0	5.2	5.2
- choose pers/soli	0.1	0.3	2.1	4.0	4.4	5.2	5.0	5.2	5.2
Panel C: 18% (Average gain: 15.1)									
Stock avoider	2.1	2.7	5.1	10.3	10.8	11.6	11.5	11.6	11.1
Procrastinator	7.6	8.9	9.4	32.7	41.6	38.8	39.0	59.1	60.7
Rational									
- no options	-1.1	-0.6	1.3	4.0	4.5	5.3	5.2	5.4	5.5
- choose IP	-0.7	-0.2	2.0	4.0	4.5	5.3	5.2	5.4	5.5
- choose pers/soli	-0.0	0.2	2.0	4.0	4.5	5.3	5.2	5.4	5.5

Table B.4: **Welfare effects on heterogeneous participants: Contributions from age 40.** The table shows percentage welfare gains from mandatory enrolment in different pension plans. The plans are all solidary, follows investment policy IP4, and have constant expected payouts through retirement. For the rational individuals, the second-to-last line in each panel reports the welfare effect when the individual is allowed, at the initial date, to replace the default investment policy IP4 with any of the specified policies IP1-IP7. In the last line in each panel, the rational individual is given the additional option to choose a personal plan ($I = 0$) instead of the default solidary feature ($I = 1$). The average gain reported is the simple average of the gains for the nine stock avoiders, the nine procrastinators, and the nine rational individuals having both options. Except for γ and ψ, the baseline parameter values listed in Table 1 are applied.

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